Structure Formation in the (Early) Universe

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Outline of 3 Lectures

1. Structure formation in the (early) universe

overview of standard cosmological model

abundance of collapsed objects at early times

2. Formation of the First Supermassive Black Holes

origin of the 10^9 $M_\odot$ black holes powering z=6 quasars discovered in Sloan Digital Sky Survey

3. Electromagnetic and Gravitational Waves Signatures of Black Hole Mergers

finding the EM counterparts and localizing SMBH binaries detected by LISA

The Standard Model of Cosmology

- **I. Cosmological Principle**

  homogeneous and isotropic on large scales
- **II. Expansion: kinematics**

  expanding in a way that preserves I.
- **III. Expansion: dynamics**

  obeys general relativity theory
- **IV. Hot Big Bang**

  hot dense state, dominated by thermal radiation
- **V. Inflation**

  brief initial phase of ‘superluminal’ expansion (?)

The Standard Model: Milestones

- **~ 80 years old** (external galaxies, expansion)
- **~ 40 yrs ago: CMB** (hot big bang, structures)
- **~ 10 yrs: Acceleration** (dark energy [?])

- **cf. Standard Model of Particle Physics**

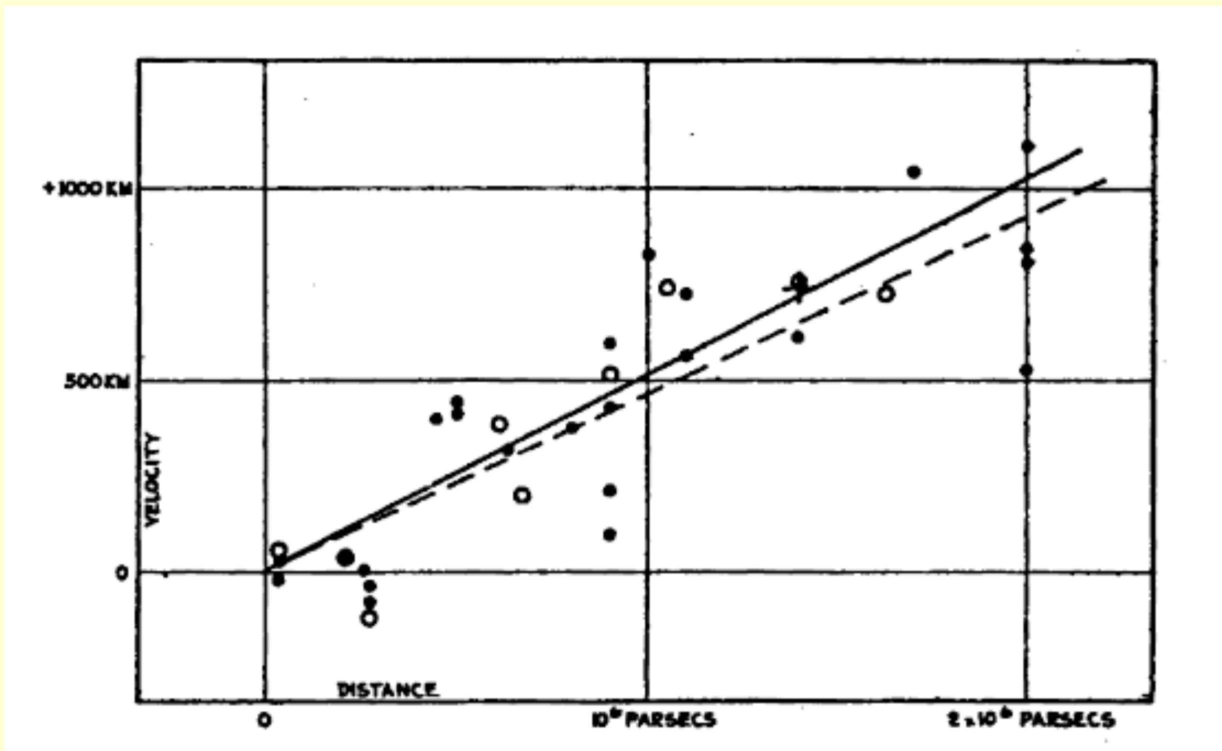
  open questions, puzzles are

  much more numerous

Expansion Kinematics

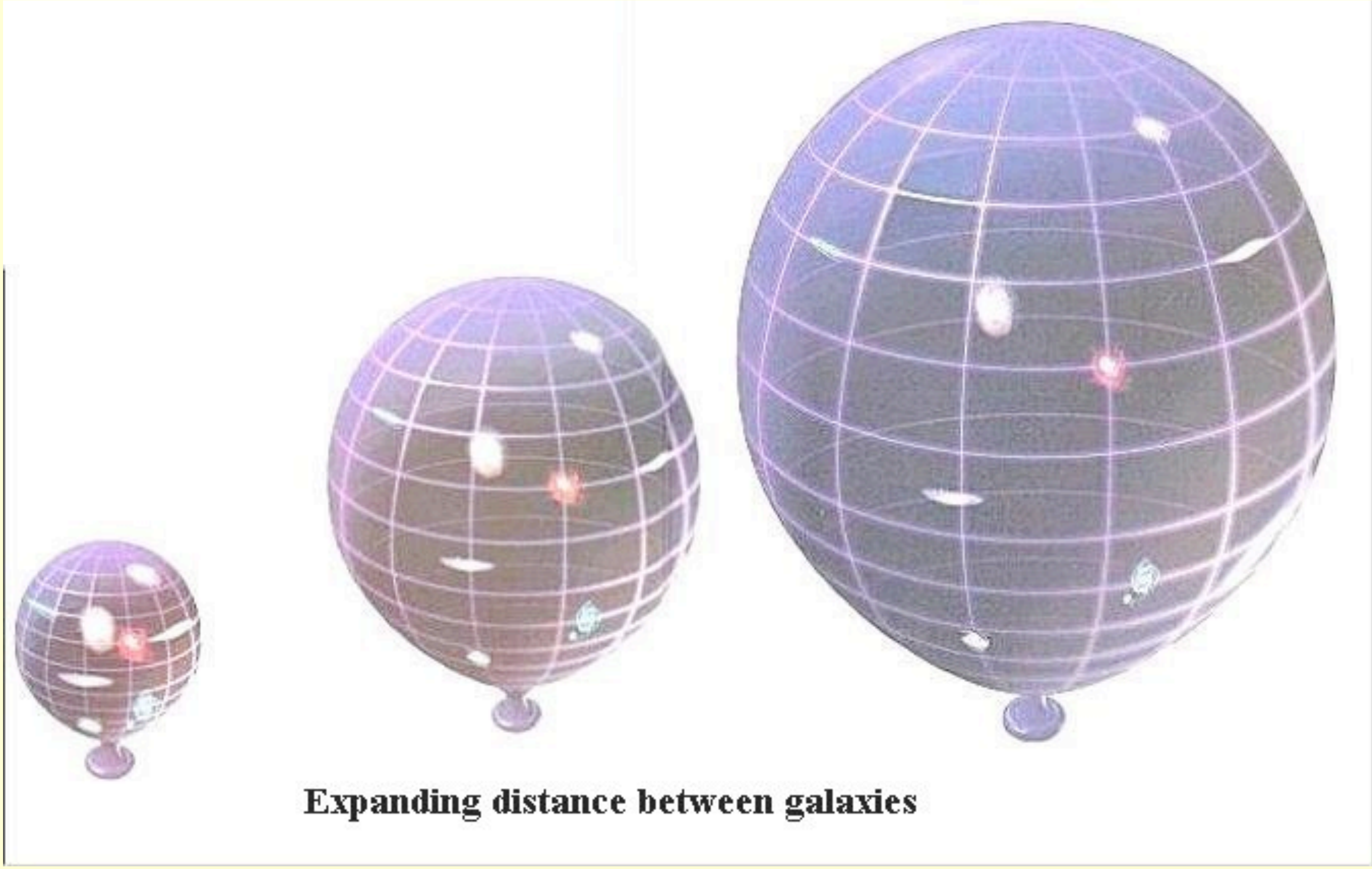
- Hubble (1929): redshift vs distance to 20 galaxies (Cepheids)
- Linear expansion $v=H_0d$ → Hubble length: d=c/H0 ~ 5 Gpc

Velocity (km/s)



Distance (1pc = 3 light years)

Expansion: Metric and Comoving Coordinates



Expansion Kinematics and FRW Metric

- General quadratic distance measure:

$$ds^2 = \sum_{\mu\nu} g_{\mu\nu} dx_\mu dx_\nu$$

- Only three metrics satisfy cosmological principle (Friedman-Robertson-Walker metric). In spherical coordinates:

$$ds^2 = c^2 dt^2 - a(t)^2 \left[\frac{dr^2}{1 - k(r/R)^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

with $k = 0, \pm 1$

- Expansion through scale factor: $H(t) \equiv \frac{1}{a}\frac{da}{dt}$

Expansion Dynamics: GR

- Evolution of FRW metric fully specified by *a(t)*
- Solution follows from Einstein's field equations

$$G_{\mu\nu} = -8\pi G T_{\mu\nu}$$

- Cosmological principle simplifies this enormously. Two independent (*00* and *ii*) components yield:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho(t) - \frac{k(c/R)^2}{a(t)^2} \quad \text{(Friedman Eqn.)}$$

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3c^2}(\rho + 3p) \quad \text{(Acceleration Eqn.)}$$

Newtonian Analog

- Energy conservation

$$E_{tot} = \frac{1}{2} m \left(\frac{dr}{dt} \right)^2 - \frac{GMm}{r(t)} = const$$

- Applied to expanding/contracting sphere of mass M

$$\left(\frac{\dot{r}}{r} \right)^2 = \frac{8\pi G}{3} \rho(t) - \frac{const}{r(t)^2}$$

- Mathematically identical to Friedmann Equation, but:

mass density ➔ energy density
total energy ➔ curvature
action-at-distance ➔ geometric theory (no forces)

Closing the Equations

- Three unknowns: $a(t), \rho(t), p(t)$ - need 3rd equation
- Equation of state relates pressure vs. energy density

$$p = w\rho$$

- Examples:

cold dark matter (non-relativistic particles):	w~0
radiation (or mass-less particles):	w = 1/3
curvature:	w = -1/3
vacuum:	w = -1

- w<-1/3 produces acceleration: “dark energy”
- **Cosmological redshift:** follows from geodesic eqn.

$$p \sim a^{-1}$$

$$\lambda_{obs} = \lambda_{em} a^{-1} = \lambda_{em}(1+z)$$

Solutions (for our universe)

- Total energy density is sum of DM, DE, CMB. In general, energy-momentum conservation requires:

$$\rho(a) = \rho_i(a=1)a^{-3(1+w)}$$

- Extrapolate backward from present-day contributions in terms of critical density $\Omega=\rho/\rho_{crit}=\rho/[3H^2 8\pi G]$:

dark energy (DE):	$w=-1$	$\Omega \sim 0.7$	$\rho \sim const$
cold dark matter (CDM):	$w \sim 0$	$\Omega \sim 0.3$	$\rho \sim a^{-3}$
radiation (CMB+neutrinos):	$w = 1/3$	$\Omega \sim 10^{-5}$	$\rho \sim a^{-4}$

- Dark energy domination is relatively recent:

  $\Omega_{DM}=\Omega_{DE}$ at $a \approx (0.3/0.7)^{1/3} \approx 0.75$ or redshift $z \approx 0.3$

- Radiation-DM transition at earlier redshift $z \approx 3000$

Growth of Inhomogeneities

- Friedmann equation + Acceleration equation + Equation of state describe global expansion of smooth (mean) background FRW universe
- Gravity enhances primordial departures from homogeneity.
- Origin of primordial density fluctuations unclear (quantum particle pair production in vacuum?)
- **Inflation:** early phase of superluminal expansion required to solve puzzles involving causality.

  Common approach is to characterize density fluctuations at the end of this "inflationary" phase.

Statistical Description of Inhomogeneities

- Full description of particle fields requires following full phase space distribution function:

$$f(\vec{x},\vec{q},t)$$

- In general, requires solving Boltzmann equations
- Often we are interested in bulk properties, such as energy density, given by low-order moments:

$$\rho(\vec{x},t) \equiv g\int \frac{d^3\vec{q}}{(2\pi)^3} f(\vec{x},\vec{q},t)$$

- Define density contrast

$$\delta(\vec{x},t) \equiv \frac{\rho(\vec{x},t) - \langle \rho(\vec{x},t) \rangle}{\langle \rho(\vec{x},t) \rangle}$$

Statistical Description of Inhomogeneities

- In linear regime, it is useful to Fourier-expand density contrast field

$$\delta(\vec{x},t) \equiv V \int \frac{d^3\vec{k}}{(2\pi)^3} \tilde{\delta}(\vec{k},\vec{q},t) e^{-i\vec{k}\cdot\vec{x}}$$

- In general, coupled evolution of δ_{DM}, δ_{γ}, δ_b, must be considered. For formation of cosmic structures, we are interested mostly in δ_{DM} (baryons follow DM).

- Power spectrum and growth function:

$$P(\vec{k},t) \equiv \left\langle \left|\tilde{\delta}(\vec{k},t)\right|^2 \right\rangle \equiv g^2(t) \left\langle \left|\tilde{\delta}(\vec{k},t=t_0)\right|^2 \right\rangle$$

Statistical Description of Inhomogeneities

- Variance in real space:

$$\sigma^2 \equiv \left\langle \left|\delta(\vec{x})\right|^2 \right\rangle = \int \frac{d^3\vec{k}}{(2\pi)^3} P(\vec{k})$$

- In isotropic case, power spectrum and variance can depend only on magnitude *k* and not on direction

$$\sigma^2 \equiv \int d\ln k \, \frac{k^3}{2\pi^2} P(k)$$

- Power per logarithmic interval in *k*

$$\Delta^2(k) \equiv \frac{k^3}{2\pi^2} P(k)$$

Statistical Description of Inhomogeneities

- Variance in a filter of size R:

$$\sigma_R{}^2 \equiv \left\langle \left|\delta(\vec{x})\right|^2 \right\rangle = \int \frac{d^3\vec{k}}{(2\pi)^3} P(\vec{k}) \left|\tilde{W}_R(\vec{k})\right|^2$$

- Common example: spherical top-hat

$$W_R(\vec{x}) = 1 \qquad if \qquad |\vec{x}| \le R$$

$$W_R(\vec{x}) = 0 \qquad if \qquad |\vec{x}| > R$$

- In *k*-space:

$$\tilde{W}_R(k) = \frac{3}{(kR)^3}\left[\sin(kR) - (kr)\cos(kr)\right]$$

- *Observations: at present, σ_{8Mpc}~1 $(M\sim10^{15}M_{\odot})$*

The Initial Power Spectrum

- Power-spectrum is "scale-invariant" if $P(k) \sim k^{-3}$ so that contribution to fluctuations is spread equally in logarithmic *k*-bins

$$\Delta^2(k) \equiv \frac{k^3}{2\pi^2} P(k) \sim const$$

- **Inflation** predicts such a scale-invariant power spectrum, if amplitude of each linear mode $\delta(k)$ is evaluated at the time when $\lambda = 2\pi/k = \lambda_H$ is the size of horizon,

$$\lambda_H = \int_0^t \frac{dt'}{a(t)} \sim H(t)^{-1}$$

- At fixed cosmic time *t*, *P(k)* is modified, as longer wavelengths enter horizon later.

Dynamical evolution of inhomogeneities

- Linear perturbations to Friedmann equation (or to Newtonian Euler equations for a fluid with overall expansion):

$$\ddot{\delta}_i + 2H(t)\dot{\delta}_i - c_{s,i}^2 a^{-2}\nabla^2\delta_i - 4\pi G\sum_j \rho_{bg,j}\delta_j = 0$$

Hubble friction

pressure gradients

gravitational sourcing

- Fourier modes δ(k) evolve independently $(\nabla^2 \rightarrow -k^2)$

Solutions

- Depends on dominant background component

$$\ddot{\delta}_i + 2H(t)\dot{\delta}_i - c_{s,i}^2 a^{-2}\nabla^2\delta_i - 4\pi G\sum_j \rho_{bg,j}\delta_j = 0$$

- For DM perturbations, ignore pressure term:

$$[\delta \sim \exp(t/t_{dyn}) \qquad (t_{dyn} = 1/\sqrt{4\pi G\rho}) \qquad Static]$$

$$\delta \sim \ln a \qquad if \qquad Rad.$$

$$\delta \sim a^1 \qquad if \qquad DM$$

$$\delta \sim a^0 \qquad if \qquad DE$$

- Expansion inhibits growth. Must follow “piece-wise” evolution through Rad/DM/DE dominated epochs

Other Effects on Growth

$$\ddot{\delta}_i + 2H(t)\dot{\delta}_i - c_{s,i}^2 a^{-2}\nabla^2\delta_i - 4\pi G\sum_j \rho_{bg,j}\delta_j = 0$$

- For baryons, pressure term is important for large k: damped oscillations on scales $\lambda < \lambda_J$ (Jeans length)

$$\lambda_J = 2\pi / k_J = \frac{2\pi c_s}{\sqrt{4\pi G\rho}} = \frac{\sqrt{\pi}c_s}{\sqrt{G\rho}}$$

- Newtonian equation fails on scales larger than the horizon $\lambda > \lambda_H$ - GR predicts growth $\delta \sim a^2$ on these scales

$$\lambda_H = \int_0^t \frac{dt'}{a(t)} \sim H(t)^{-1}$$

Free Streaming of DM particles

- By cosmic time *t*, dark matter particles travel a distance

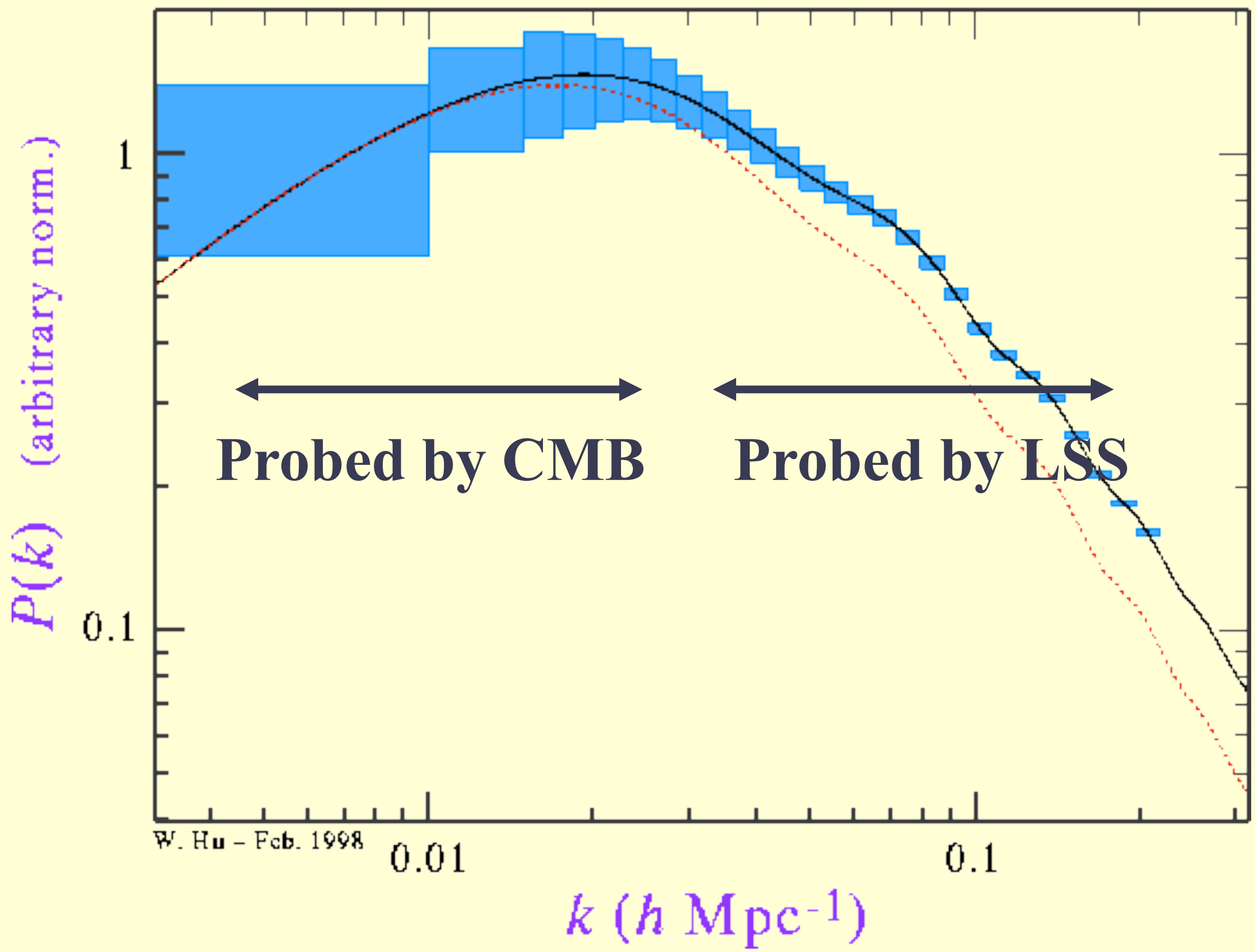
$$\lambda_{FS} = \int_0^t \frac{v(t')dt'}{a(t')} = \frac{a(t)}{a(t_{nr})} 2t_{nr} \left[\frac{5}{2} + \ln \frac{a(t_{RM})}{a(t_{nr})} \right]$$

- Free streaming scale depends on the time t_{nr} when particle becomes non-relativistic due to decaying momentum ➔ depends on particle mass m_{DM}

- Logarithmic correction from change in background evolution a(t) at transition from Rad. to DM domination

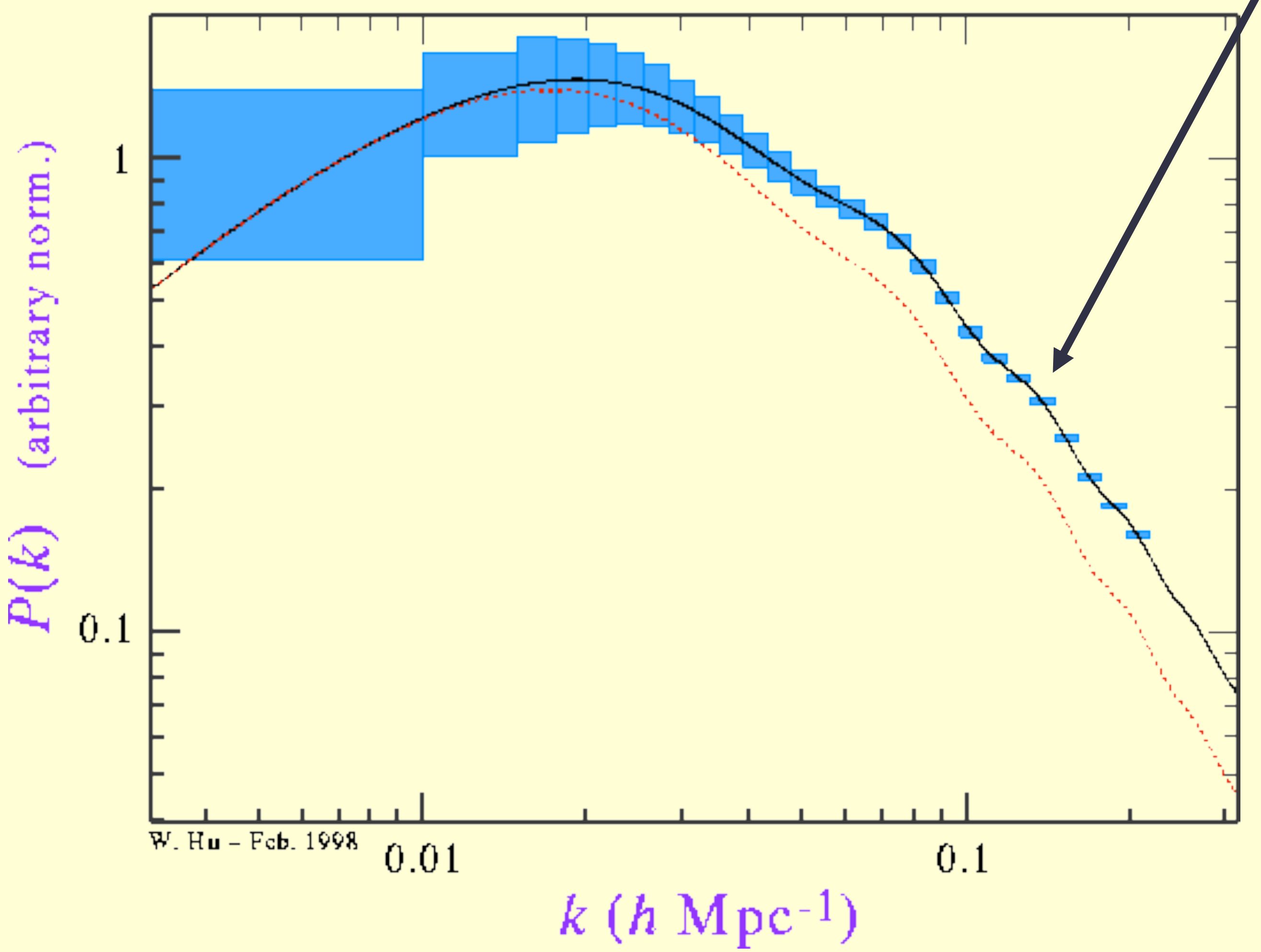
$$\lambda_{FS} = 0.5\left(\Omega_{DM} h^2\right)^{1/3} \left(\frac{m_{DM}}{1keV}\right)^{-4/3} Mpc$$

Linear DM power spectrum at z=0



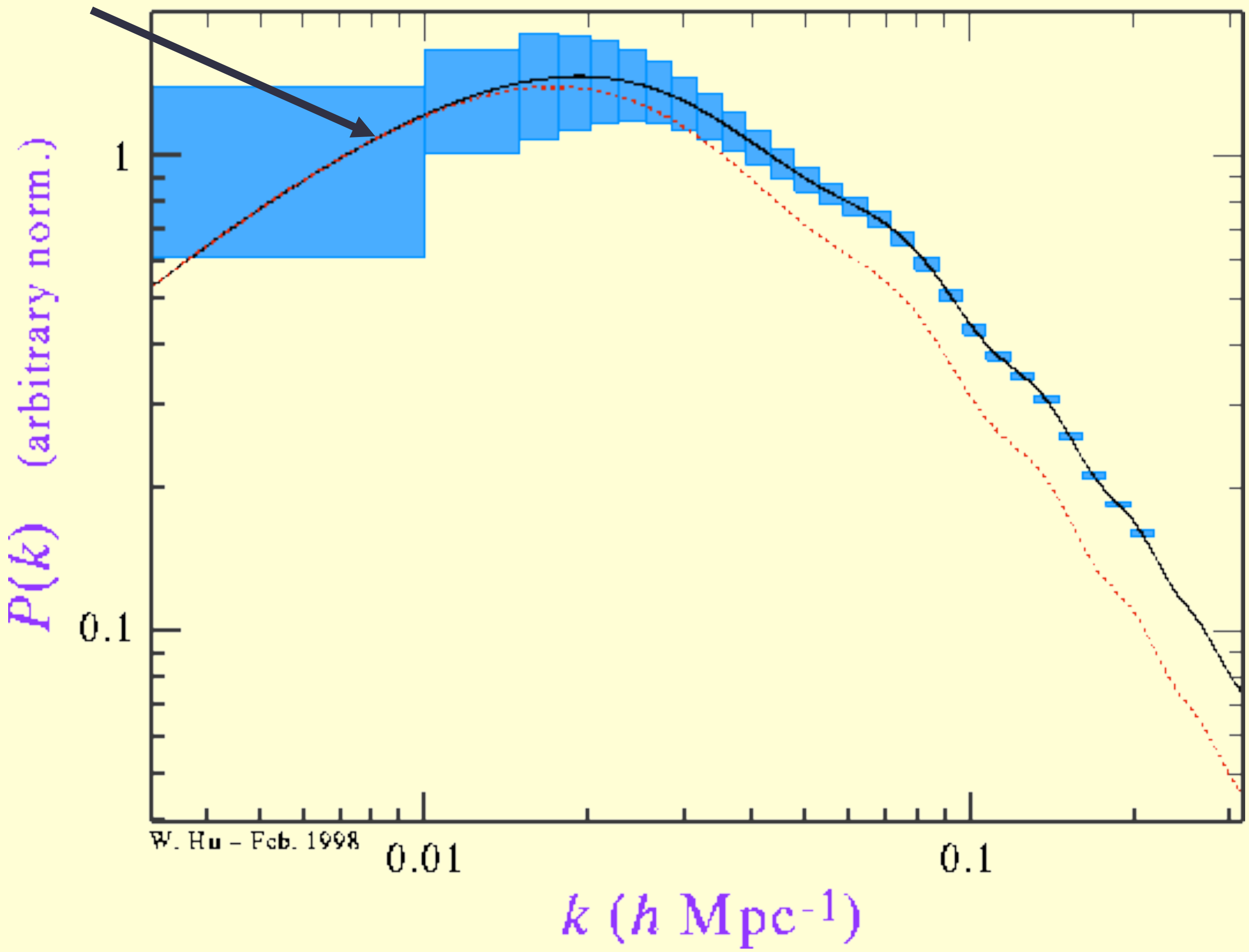
Linear DM power spectrum at z=0

Small scales enter horizon at $t < t_{RM}$ and stall; still show $P(k) \sim k^{-3}$

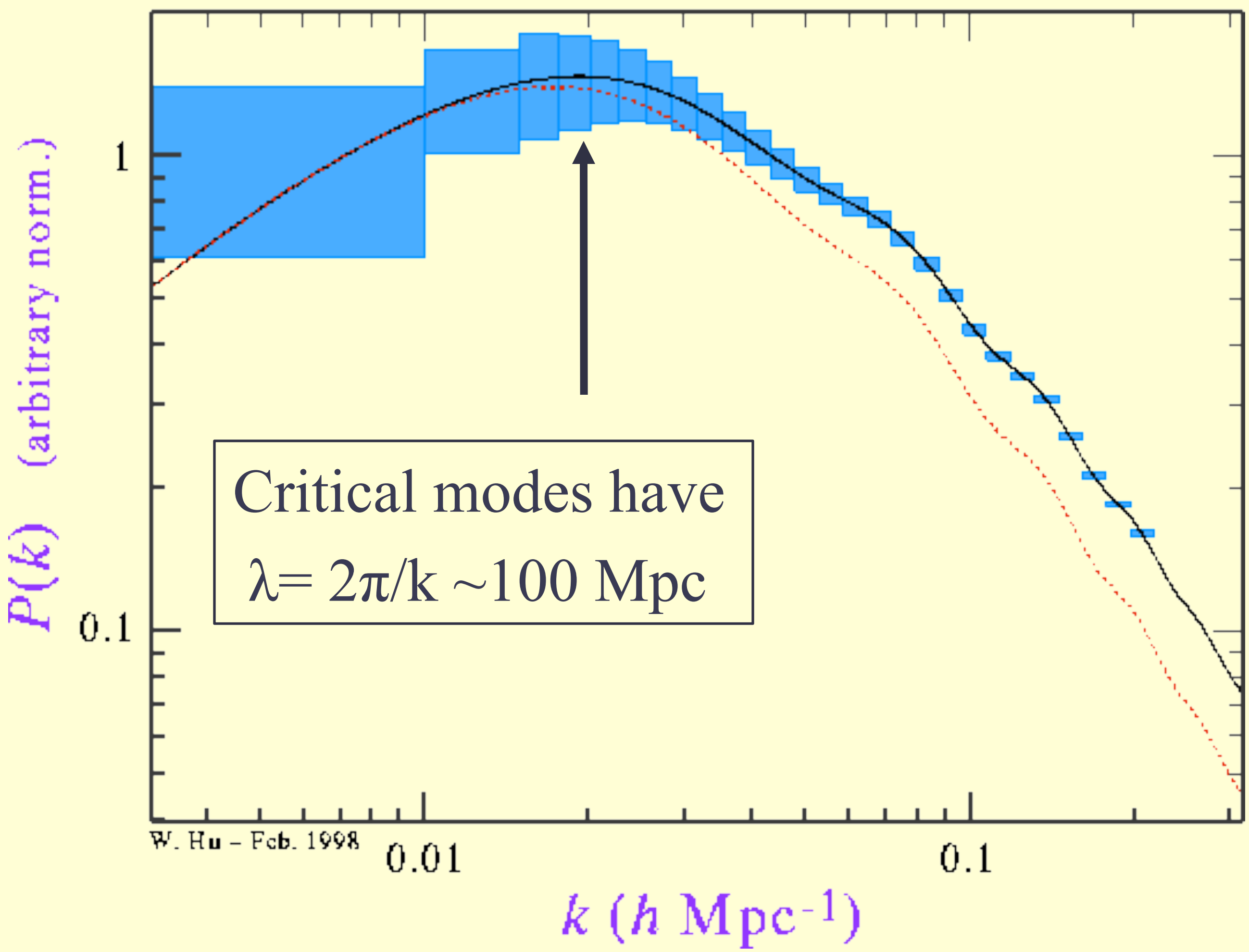


Linear DM power spectrum at z=0

Large scales enter horizon at $t > t_{RM}$ and have grown only since; $P(k) \sim k^{-1}$



Linear DM power spectrum at z=0



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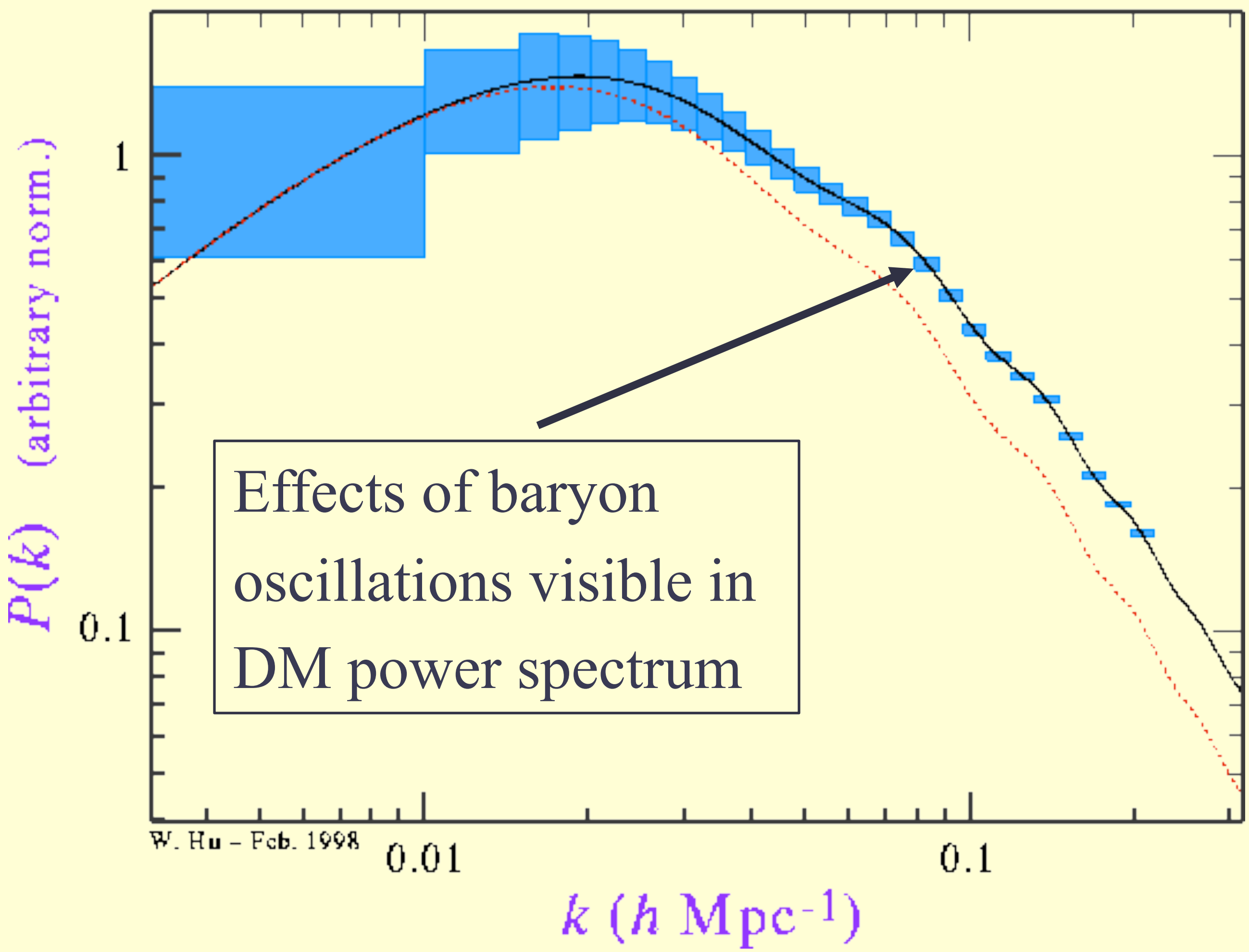

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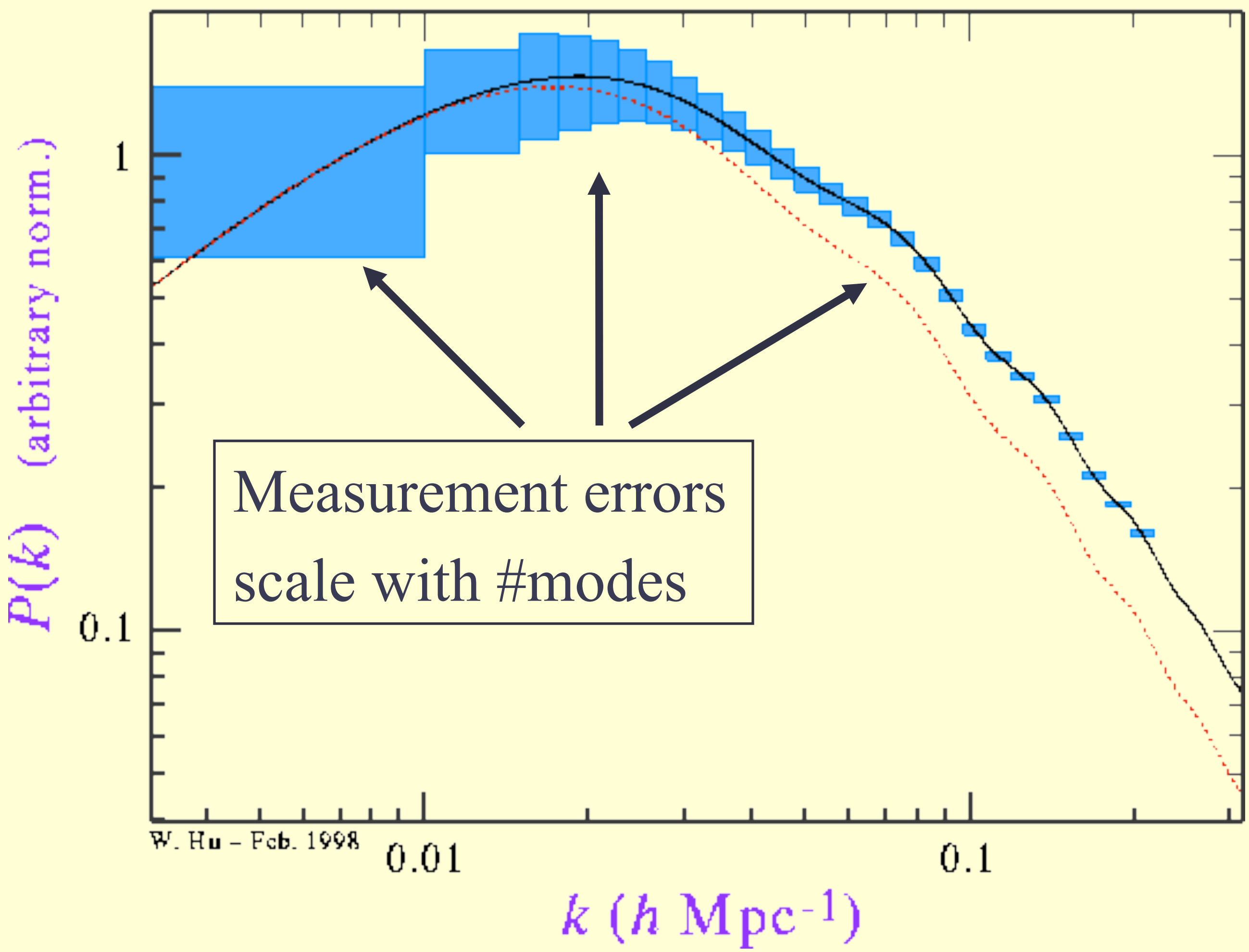

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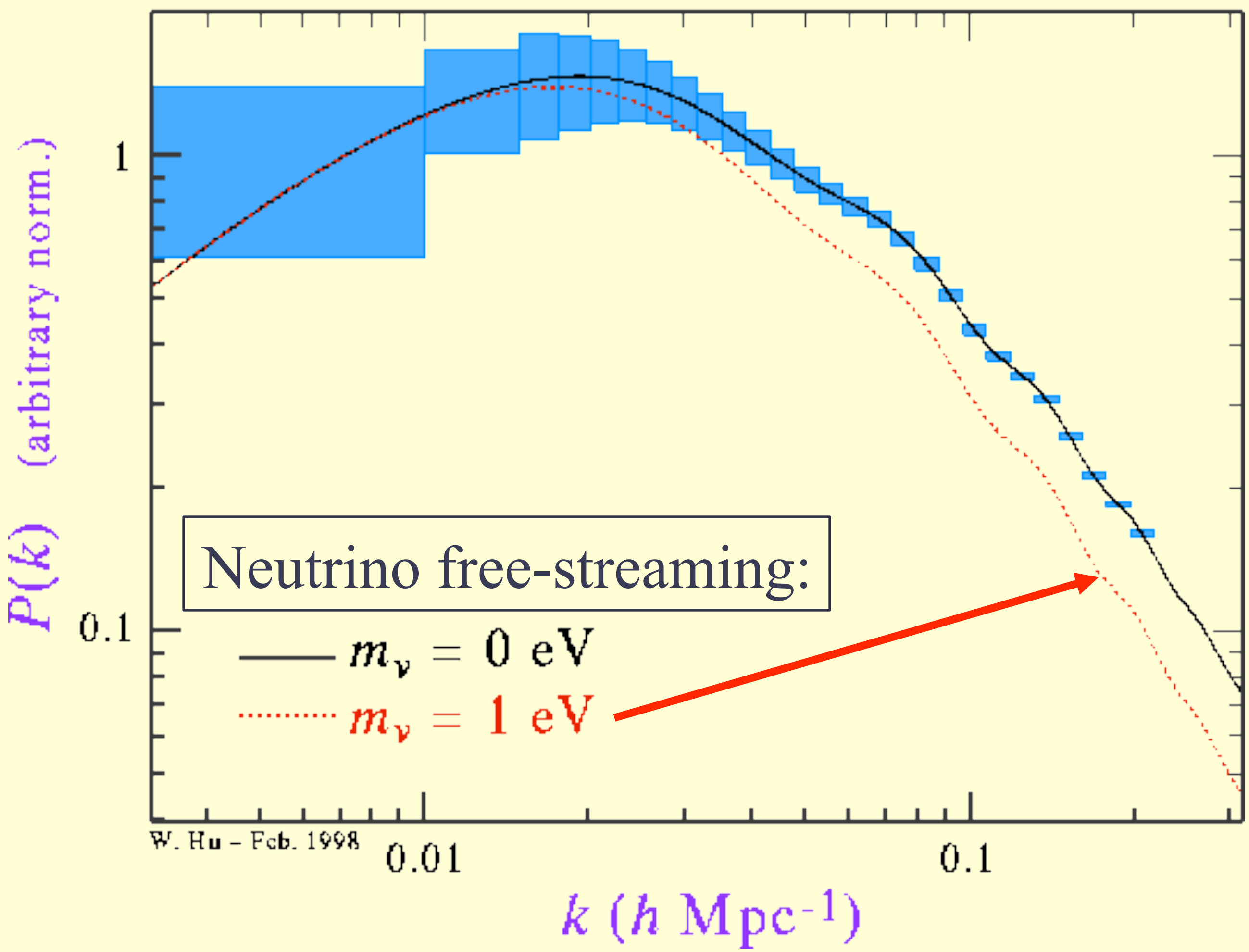
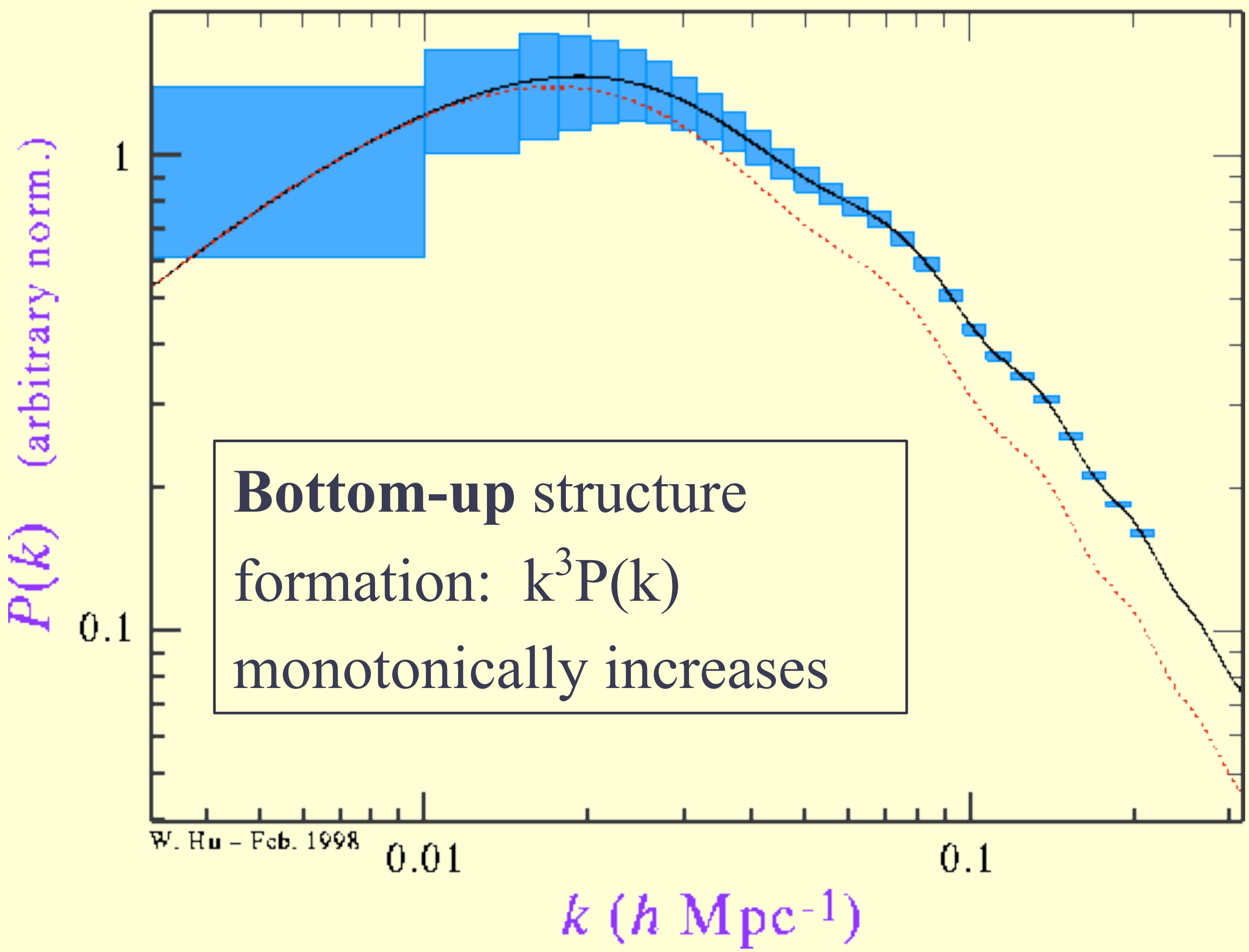


Linear DM power spectrum at z=0



Bottom-up structure formation: $k^3P(k)$ monotonically increases

Nonlinear regime

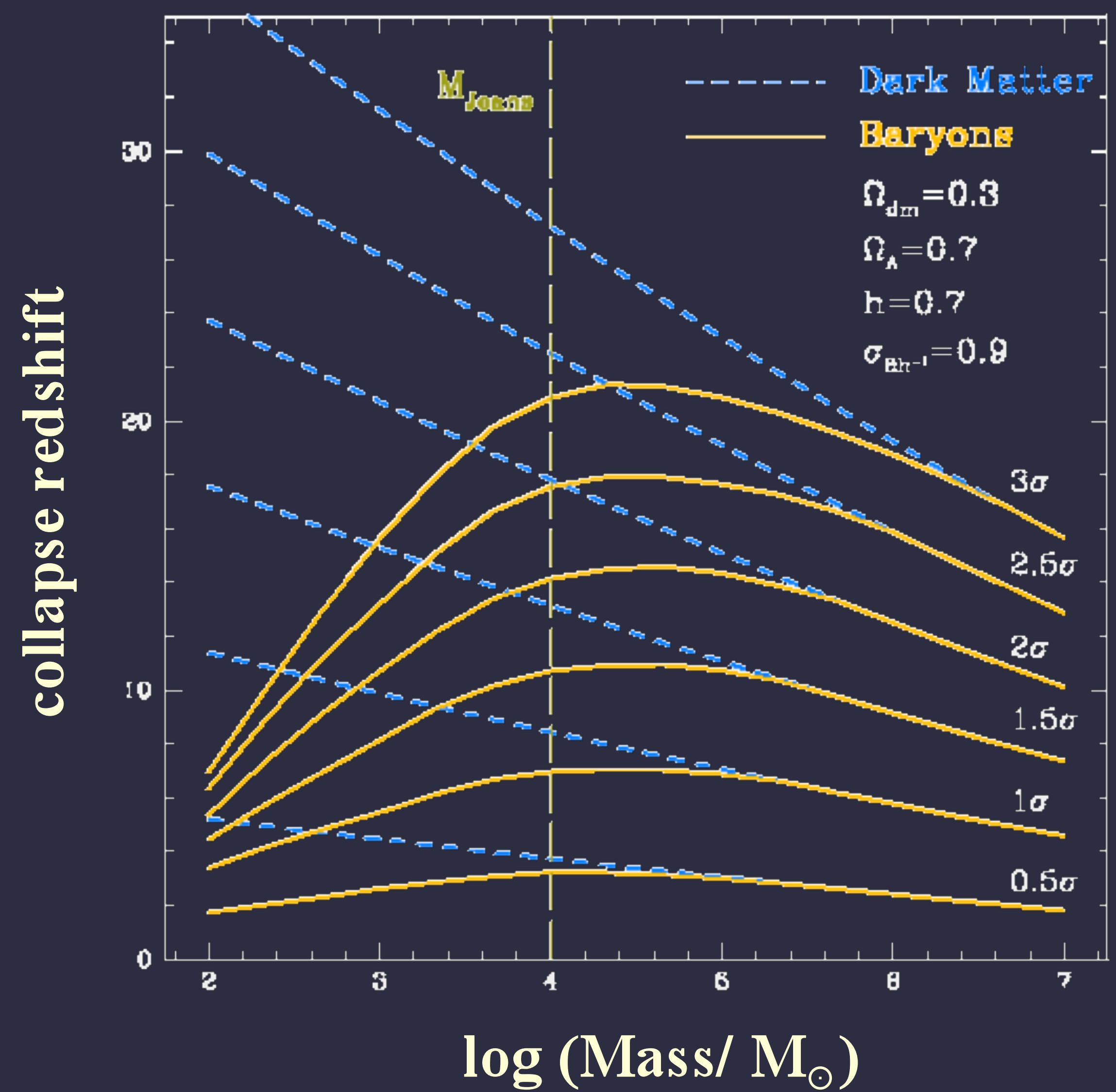
- Large scales, safely in linear regime, are best to study cosmology
- For astrophysically interesting objects, we need to study non-linear regimes
- Recall: $\sigma^2_R(t) \sim g^2(t)\, \sigma^2_R(t=t_0)$. For $P(k) \sim k^n$

$$\sigma_R{}^2 = \int \frac{d^3\vec{k}}{(2\pi)^3} P(\vec{k}) \left|\tilde{W}_R(\vec{k})\right|^2 \sim R^{-(n+3)}$$

$$\sigma_M = const \times M^{-\frac{n+3}{6}}$$

- At a given time, there is a characteristic mass-scale of non-linearity, M^*, where $\sigma_{M^*}=1$. M^* grows w/ time.

Nonlinear Scales at Early Times



Z. Haiman
PhD thesis 1998

Jeans length for Baryons

- In general, Jeans mass:

$$M_J \equiv \frac{4\pi}{3}\left(\frac{\lambda_J}{2}\right)^3 \rho = const\,\frac{T^{3/2}}{\rho^{1/2}}$$

- Depends on evolution of background gas temperature T_b. At $z>z_{crit}\approx150$, Compton scattering with CMB photons keeps $T_b=T_{CMB}\sim(1+z)$, and

$$M_J = 1.35\times10^5\left(\frac{\Omega_{DM}h^2}{0.15}\right)^{-1/2} M_{sun} = const.$$

- At $z<z_{crit}\approx150$, gas decouples thermally from CMB, and temperature evolves adiabatically, $T_b\sim(1+z)^2$

$$M_J = 4.54\times10^3\left(\frac{\Omega_{DM}h^2}{0.15}\right)^{-1/2}\left(\frac{1+z}{10}\right)^{3/2}$$

Spherical Collapse

- Consider spherical top-hat perturbation in a smooth FRW background universe, with initial radius R_i and density $\rho_i=(1+\delta_i)\rho_{bg}$
- Evolution of $R(t)$ identical to that of $a(t)$ in a denser FRW universe
- Parametric solution available in DM-only universe:

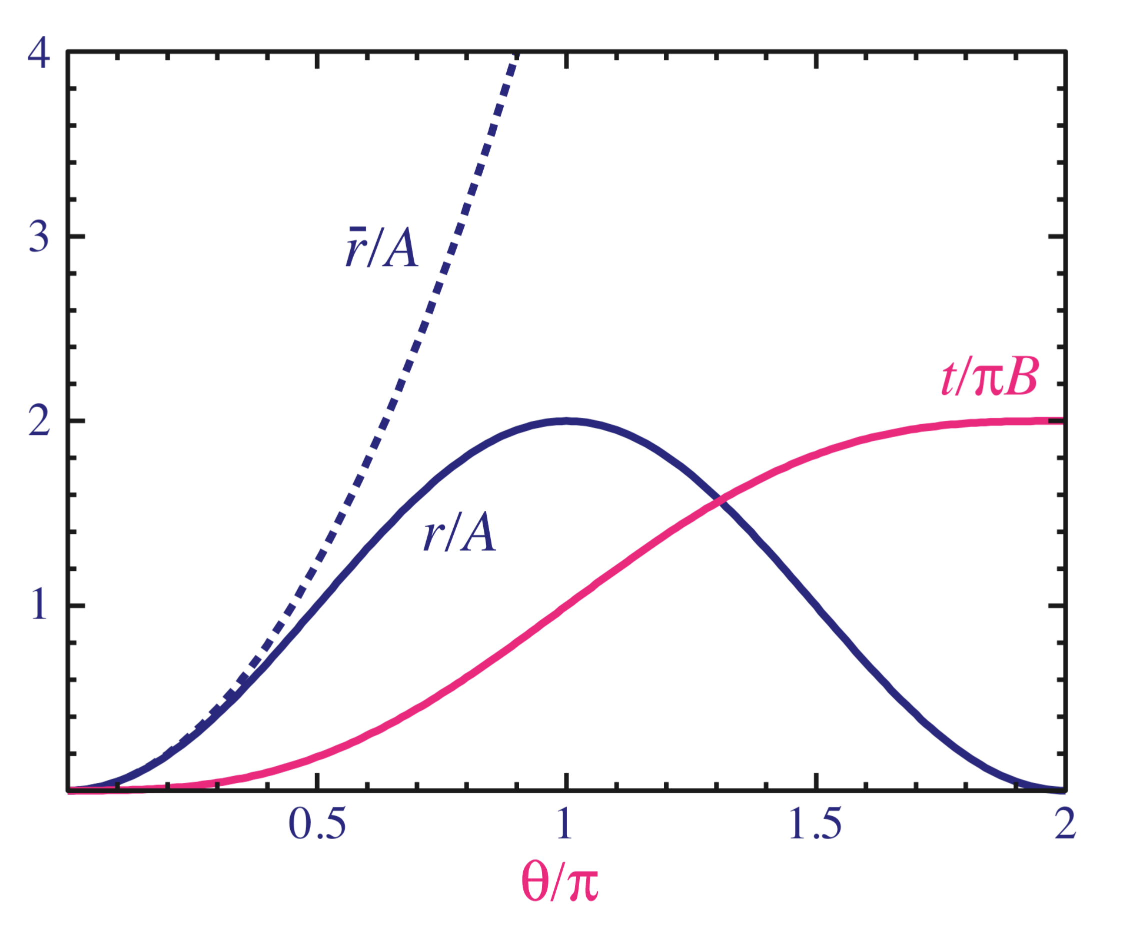
$$R(\theta)=A(1-\cos\theta)$$

$$t(\theta)=B(\theta-\sin\theta)$$

with $A=R_i(1+\delta_i)/[2(\delta_i-(\Omega_m^{-1}-1)]$

$B=(1+\delta_i)/2H_i\Omega_m^{1/2}[(\delta_i-(\Omega_m^{-1}-1)]^{3/2}$

Spherical Collapse



W.Hu cosmology lecture notes

Overdense region lags behind

Turnaround at $\theta=\pi$

Time-symmetric recollapse at $\theta=2\pi$

Properties of Spherical Top Hat

- Density evolution: $$\delta = \frac{9}{2}\frac{(\theta - \sin\theta)^2}{(1-\cos\theta)^3} - 1$$

- At turn-around, $\delta = 9\pi^2/16 - 1 \approx 4.6$

- Perturbation cannot collapse to a point. DM shells cross; gas shocks convert infall to thermal motions.

- Virialization must occur at $r_{vir} \approx r_{max}/2$:

  $E_{pot} = -2E_{kin}$ in virial equilibrium; $E_{pot} = GM/r$

  $E_{tot} = E_{pot}(r_{max}) = 1/2\ E_{pot}(r_{vir})$

  (turnaround) (after virialization)

- Density contrast $\delta_{vir} = \rho(\theta=3\pi/2)/\rho_{bg}(\theta=2\pi) = 18\pi^2 \approx 178$

Mass Function of Collapsed Halos

- End result of spherical collapse is a virialized halo
- "NFW" density profile $\rho \sim r^{-1}$ (core) - r^{-3} (edge) from N-body simulations (Navarro, Frenk & White 1997)
- Collapse occurs at $\theta=2\pi$, when linearly extrapolated overdensity reaches $\delta_L=(3/5)(3/4)^{2/3}(2\pi)^{2/3}=1.686$
- Very powerful concept: combine with initial Gaussian random fluctuations, the abundance of nonlinear halos can be computed from linear theory
- Originally by Press-Schechter – vindicated by N-body simulations to within factor of ~two

Mass Function of Collapsed Halos

- Consider initial density field, extrapolated to $z=0$ with linear theory, smoothed on mass-scale M
- Probability distribution $P(\delta)$ of overdensity δ at a random point is a Gaussian with variance $\sigma^2(M)$
- Associate fraction $F(>M)$ of mass residing in collapsed halos with mass *M or larger* with regions that have $\delta \geq \delta_{crit} = 1.686$:

$$F(>M) = \frac{1}{\sqrt{2\pi}\sigma_M} \int_{\delta_{crit}}^{\infty} d\delta \exp\left(-\frac{\delta^2}{2\sigma_M^2}\right)$$

$$= \frac{1}{2} erfc\left(\frac{\delta_{crit}}{\sqrt{2}\sigma_M}\right)$$

Mass Function of Collapsed Halos

- To find space density of halos with mass *M:*

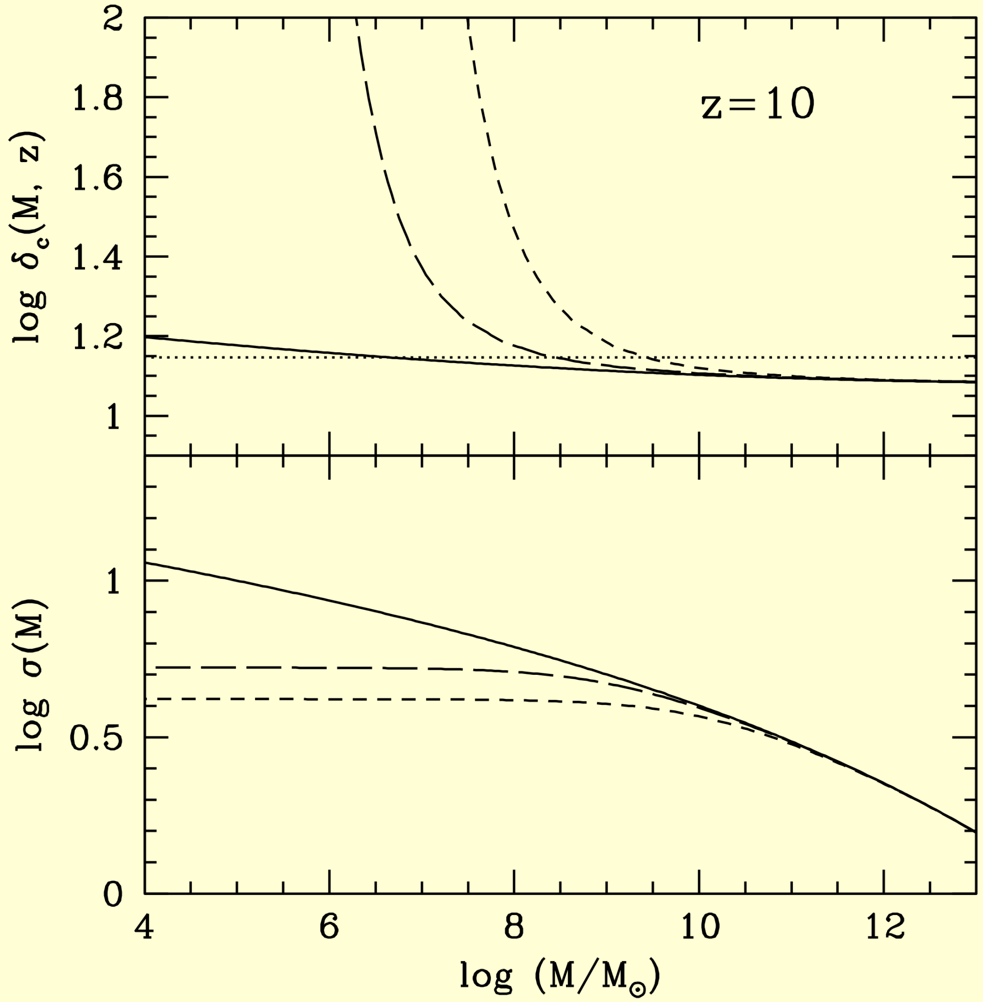
$$\frac{dN}{dM} = \frac{\rho}{M}\frac{dF(>M)}{dM} = -\sqrt{\frac{2}{\pi}}\frac{\rho}{M}\frac{\delta_{crit}}{\sigma_M}\frac{d\ln\sigma_M}{dM}\exp\left(-\frac{\delta_{crit}^2}{2\sigma_M^2}\right)$$

- Integrates to unity as M → 0
- dN/dM → M^{-2} as M → 0
- dN/dM → exp(-*const* $M^{2/3}$) as M → ∞
- Characteristic mass defined by $\sigma_{M*} \approx \delta_{crit}$ close to nonlinear mass scale defined earlier

Mass Function vs. N-body Simulations

(Hu & Kravtsov 2002)

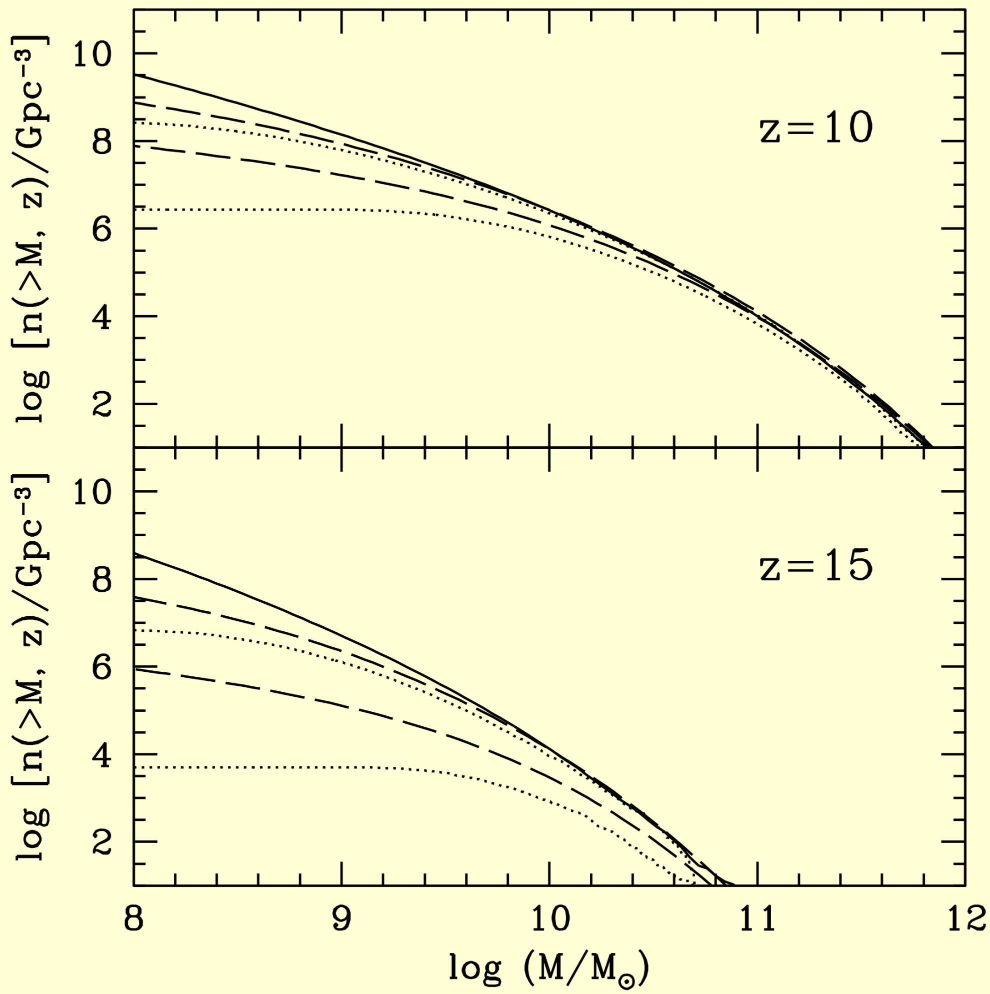
Example Mass Functions: Thresholds



CDM
2 keV WDM
1 keV WDM

(Mesinger, Perna and Haiman 2005)

Example Mass Functions



CDM
2 keV WDM
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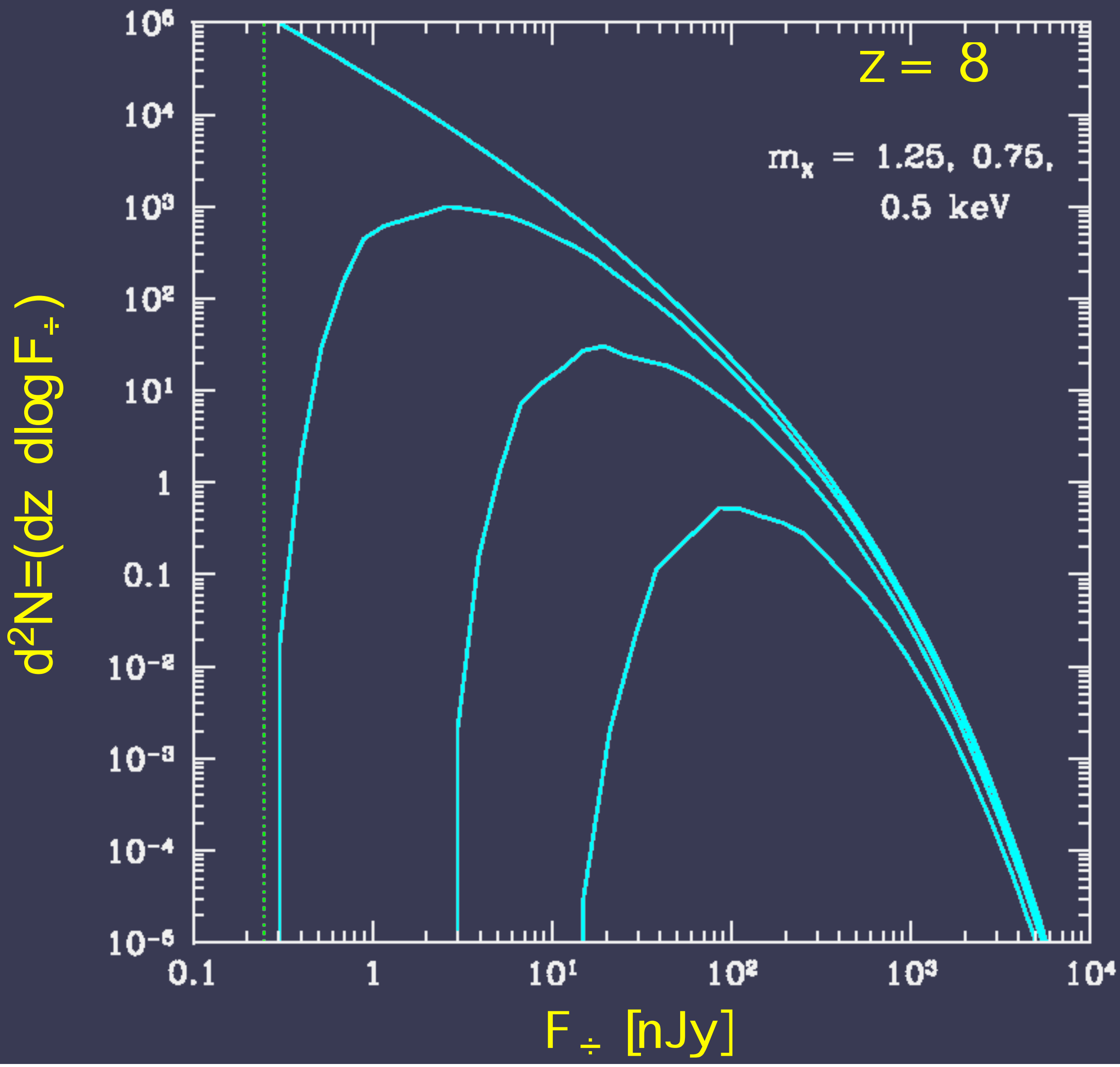
(Mesinger, Perna and Haiman 2005)

Possible Implications of WDM for High-z

- Difficult to produce photons to re-ionize the Universe
- Difficult to produce *z*>*6* quasars / galaxies
- Difficult to produce z>8.2 Gamma Ray Bursts
- Possible cut-off in future JWST luminosity function

Luminosity Function



Barkana, Haiman, Ostriker (2001)