Lecture 3 : Elliptical galaxies II

1 Shapes of elliptical galaxies

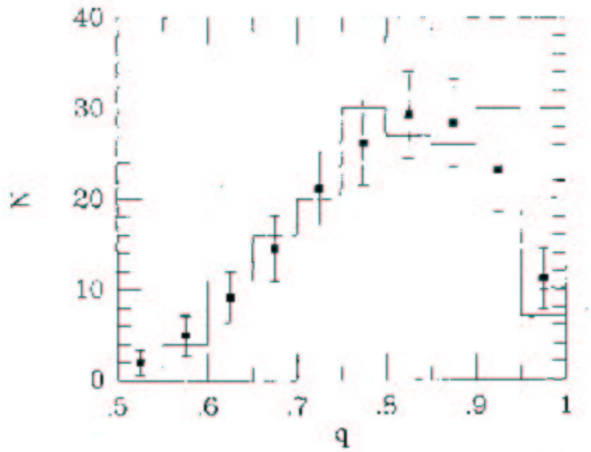

Figure 1: Observed distribution of apparent axial ratios (Ryden, 1992, ApJ, 396, 445)

The observed ellipticity of a galaxy is a combination of its true shape and the projection effects. The ellipticity of a galaxy, defined as $\varepsilon = 1 - b/a$, is therefore not intrinsic. We would like to know the true shape of ellipticals, disentangling the projection effects somehow. For ellipsoidal objects, there are in general three axes. For example, the surfaces of constant density can be ellipsoidal, meaning,

$$\frac{d^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = r^2 \,, \tag{1}$$

where a, b, c can be functions of r. An ellipsoidal is called oblate if $a = b > c$, prolate if $a > b = c$ and triaxial if $a > b > c$.

The distribution of the observed axial ratios, $N(b/a)$ shows a rise from E0 to E2 followed by a decrease. One can ask if this distribution can be explained by random orientation of either oblate or prolate ellipsoidals. It is in general difficult with only prolates or oblates, especially the rising part. One can fit the distribution with a distribution of triaxials, with $a : b : c \sim 1 : 0.95 : 0.7$ (which is close to being oblate) with Gaussian dispersion of ~ 0.2. That ellipsoidals are in general triaxials is supported by the *isophote twists* seen in many ellipticals. The position angle of the major axis of isophotes (contours of equal surface brightness) changes with radius, which cannot the result of projection of oblate or prolate objects. It can occur if objects are triaxial with axial ratios changing with radius.

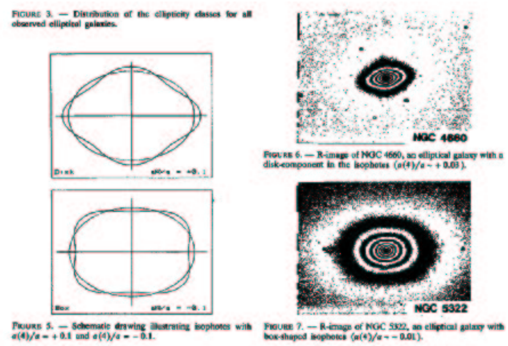

Figure 2: Examples of boxy and disky isophotes from Bender et al(1988).

Since the isophotes are exact ellipses, one can express them as a Fourier series,

$$R(\phi) = a_0 + \sigma a_n \cos(n\phi) + \Sigma b_n \sin(n\phi)\,, \tag{2}$$

where $R(\phi)$ is the ellipse that fits an isophote. The coefficient a_4 carries information of the shape of the isophotes, negative values make a boxy ellipse whereas positive values make a disky isophote.

2 Theoretical interlude II : Virial theorem

If we take the first moment of the Boltzmann equation in velocity, we would get,

$$\int \frac{\partial f}{\partial t} v_j d^3v + \int v_i v_j \frac{\partial f}{\partial x_i} d^3v - \frac{\partial \phi}{\partial x_i} \int v_j \frac{\partial f}{\partial v_i} d^3v = 0\,. \tag{3}$$

But

$$\int\int\int v_j \frac{\partial f}{\partial v_i} d^3v = \int\int v_j (f(v_i=\infty) - f(v_i=-\infty)) d^2v_{\neq i} - \int\int\int \frac{\partial v_j}{\partial v_i} f d^3v = 0 - \delta_{ij} n\,, \tag{4}$$

since $f(v_i = \pm\infty) = 0$, which gives us,

$$\frac{\partial}{\partial t}(n\bar{v_j}) + \frac{\partial}{\partial x_i}(n\bar{v_i v_j}) + n\frac{\partial \phi}{\partial x_j} = 0\,. \tag{5}$$

This is the 2nd Jeans equation (2JE). One obtains the tensor Virial theorem from this equation by $m\int x_k[2JE]d^3x$, where m is the mass of an individual object and $mn = \rho$. We get,

$$\int x_k \frac{\partial(\rho\bar{v_j})}{\partial t} d^3x = -\int x_k \frac{\partial}{\partial x_i}(\rho\bar{v_i v_j})d^3x - \int x_k \rho \frac{\partial \phi}{\partial x_j} d^3x\,. \tag{6}$$

The last term is the potential energy tensor,

$$\begin{aligned} W_{jk} &= -\int x_j \rho \frac{\partial \phi}{\partial x_k} d^3x = G\int\int x_j \rho(\mathbf{x})\rho(\mathbf{x}')\frac{(x'_k - x_k)}{|\mathbf{x}'-\mathbf{x}|^3} d^3x d^3x' \\ &= -G\int\int x'_j \rho(\mathbf{x}')\rho(\mathbf{x})\frac{(x'_k - x_k)}{|\mathbf{x}'-\mathbf{x}|^3} d^3x' d^3x \end{aligned} \tag{7}$$

where we have exchanged the variables $\mathbf{x}'$ and $\mathbf{x}$, and used $\phi = -G\int \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x'$. Adding and diving by 2, we get,

$$W_{jk} = -\frac{G}{2}\int\int \rho(\mathbf{x})\rho(\mathbf{x}')\frac{(x'_k - x_k)(x'_j - x_j)}{|\mathbf{x}'-\mathbf{x}|^3} d^3x' d^3x \tag{8}$$

We therefore have $W_{jk} = W_{kj}$. The trace of this tensor is given by,

$$\begin{aligned} Trace(W_{jk}) &= -\frac{G}{2}\int\int \rho(\mathbf{x})\rho(\mathbf{x}')\frac{\Sigma(x'_k - x_k)^2}{|\mathbf{x}'-\mathbf{x}|^3} d^3x' d^3x \\ &= -\frac{G}{2}\int\int \frac{\rho(\mathbf{x})\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x' d^3x \\ & \frac{1}{2}\int \rho\phi d^3x = W\,, \end{aligned} \tag{9}$$

which is the potential energy.

Consider the second derivative (in time) of the moment of inertia tensor, $I_{jk} = \int \rho x_j x_k d^3x$,

$$\frac{dI_{jk}}{dt} \quad = \int \frac{d\rho}{dt} x_j x_k d^3$$

$$
\begin{aligned}
&= -\int \frac{\partial(\rho\bar{v_i}}{\partial x_i} x_j x_k d^3x \\
&= \int \rho\bar{v_i}(\delta_{ij}x_k + \delta_{ik}x_j)d^3x \\
&= \int \rho(\bar{v_j}x_k + \bar{v_k}x_j)d^3x\,,
\end{aligned} \tag{10}
$$

where we have first used the continuity equation ($\frac{\partial n}{\partial t} + \frac{\partial}{\partial x_i}(n\bar{v_i}) = 0$) and then the divergence theorem ($\int_V g\nabla\mathbf{F}d^3x = \int_{dV} g\mathbf{F}d^2\mathbf{s} - \int_V(\mathbf{F}\cdot\nabla)gd^3x$). Finally, we have,

$$
\frac{d^2}{dt^2}I_{jk} = \int \left[x_k\frac{\partial}{dt}(\rho\bar{v_j})x_j\frac{\partial}{\partial t}(\rho\bar{v_k})\right]d^3x\,, \tag{11}
$$

showing that the first term in equation 6

$$
\int x_k\frac{\partial(\rho\bar{v_j}}{\partial t}d^3x = \frac{d^2}{dt^2}\frac{1}{2}I_{jk}\,. \tag{12}
$$

The second term can be transformed using the divergence theorem again to,

$$
-\int x_k\frac{\partial}{\partial x_i}(\rho v_i\bar{v}_j)d^3x = -\int_{dV}x_k\rho v_i\bar{v}_j d^2x_{\neq i} + \int_V \rho v_i\bar{v}_j\delta_{ik}d^3x = \int_V \rho v_k\bar{v}_j d^3x\,, \tag{13}
$$

which is the kinetic energy tensor, $2K_{kj}$. This tensor can be written (with $v_k\bar{v}_j = \bar{v_k}\bar{v_j} + \sigma^2_{kj}$) as $K_{kj} = \int\frac{1}{2}\rho\bar{v_k}\bar{v_j}d^3x + |\int\frac{1}{2}\rho\sigma^2_{kj}d^3x = T_{jk} + \Pi_{jk}$. The first term denotes ordered motion and the second term (dispersion tensor) represents random motion. The dispersion tensor is defined as $\sigma^2_{ij} = \langle(v_i - \bar{v_i})(v_j - \bar{v_j})\rangle = \langle v_iv_j\rangle - \langle v_i\rangle\langle v_j\rangle$. The trace of this tensor is $\int\frac{1}{2}\rho(\sigma^2 11 + \sigma^2 22 + \sigma^2 33)d^3x$ which is the total kinetic energy. We therefore have the **tensor virial theorem**,

$$
\frac{1}{2}\frac{d^2}{dt^2}I_{ik} = 2T_{ik} + 2\Pi_{ik} + W_{ik}\,. \tag{14}
$$

The trace, in the equilibrium case, $\frac{d^2}{dt^2} = 0$, reduces to $2T + 2\Pi = 2K = -W$, which is the scalar virial theorem.

Let us consider an application of the tensor virial theorem. For an axisymmetric elliptical, with the z-axis as the axis of symmetry, one has $W_{ij} = \Pi_{ij} = T_{ij} = 0$ (for $i \neq j$), and $W_{xx} = W_{yy}$, $\Pi_{xx} = \Pi yy$ and $T_{xx} = T_{yy}$. The only non-trivial equations are,

$$
2T_{xx} + 2\Pi_{xx} + W_{xx} + 0\,; 2T_{zz} + 2\Pi_{zxz} + W_{zz} = 0\,. \tag{15}
$$

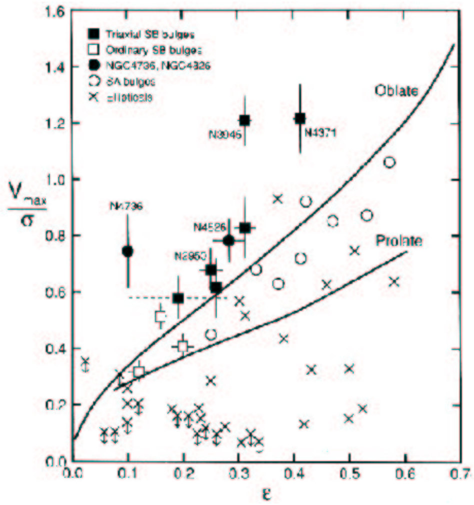

Figure 3: Relation of v_0/σ_0 with ellipticity (Kormendy 1982).

Which implies that,

$$\frac{2T_{xx} + 2\Pi_{xx}}{2T_{zz} + 2\Pi_{zz}} = \frac{W_{xx}}{W_{zz}} \sim a/b \tag{16}$$

If the system is rotationally flattened along z-axis, then W_{zz} would be smaller (since the pairs $(z - z')$ would be smaller than $(x - x')$) than W_{xx}, and approximately depend on the ratio of the major to minor axis of the rotation ellipsoid (a along the x-axis and b along z-axis). The last equality is very crude. For a rotationally flattened system, there is only streaming motion around the z-axis, and $2T_{xx} + 2T_{yy} = \int \rho \bar{v}_\phi^2 d^3x = MV_0^2$, where v_0 is the mass weighted rotation velocity, and $T_{zz} = 0$. If σ_0^2 is the mass weighted mean square random velocity along line of sight, then we also have $2\Pi_{xx} = M\sigma_0^2$. This means that,

$$\frac{v_0}{\sigma_0} \sim \sqrt{2(a/b) - 2} = \sqrt{(1 - b/a)/b/a} = \sqrt{\varepsilon/(1 - \varepsilon)}. \tag{17}$$

One finds that for luminous ellipticals $(v_0/\sigma_0)/\sqrt{\varepsilon/(1-\varepsilon)} < 1$ and so they are *not* rotationally flattened—they are flattened by velocity anisotropy, which further points towards triaxiality. Lower luminosity ellipticals are rotationally flattened. Interestingly, v_0/σ_0 corelates well with a_4/a which means that disky galaxies rotate and boxy galaxies do not.

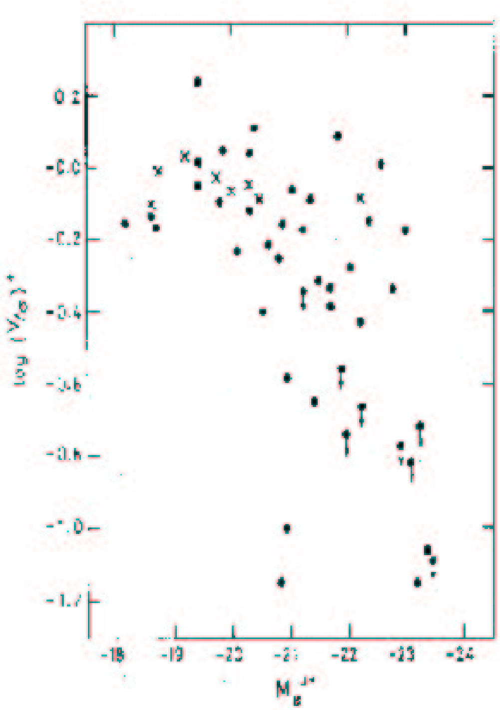

Figure 4: Relation of v_0/σ_0 with blue luminosity (Davies et al 1983, ApJ, 266, 41).

3 X-rays from ellipticals

Massive ellipticals often emit thermal X-rays (free-free) from a X-ray corona. The X-ray luminosities of order $10^{40}-10^{42}$ erg/s, with $T\sim 10^7$ K and $n_e\sim 0.1-10^{-4}$ per cc. The total amount hot gas implied is of order 10^8-10^{10} solar mass. The implied cooling time is very short 10^8-10^9 yrs, so that one needs heating sources to replenish the energy lost through cooling. SNs are probably the sources of heat. The relation between L_x and L_B has however a large scatter which is not well understood.

X-ray observations allow one to determine the potential well of the galaxy and therefore the mass, by assuming hydrostatic equilibrium. Assuming $dp/dr = -GM(<r)\rho/r^2$, one can write,

$$M(<r) = r\frac{kT}{G|mu}[-\frac{d\ln\rho}{d=lnr}-\frac{d\ln T}{d\ln r}] \tag{18}$$

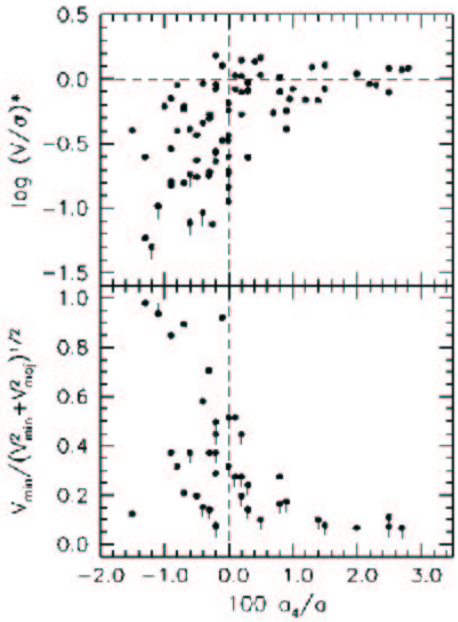

Figure 5: Relation of v_0/σ_0 with a_4 (Kormendy & Bender 1996).

The term in bracket is very close to 2 for most cases, and so one has,

$$M(<r) \sim 4\times 10^{11} M_\odot (T/10^7 K)(r/10\,kpc)\,. \tag{19}$$

4 Dark matter in dwarf galaxies

An interesting topic of research at present is the dominance of dark matter in dwarf galaxies. Very low luminosity dwarg spheroidals seem to have a very large ratio of M/L. In fact there is a correlation between the visual magnitude and the M/L ratio for the Local Group dSph galaxies.

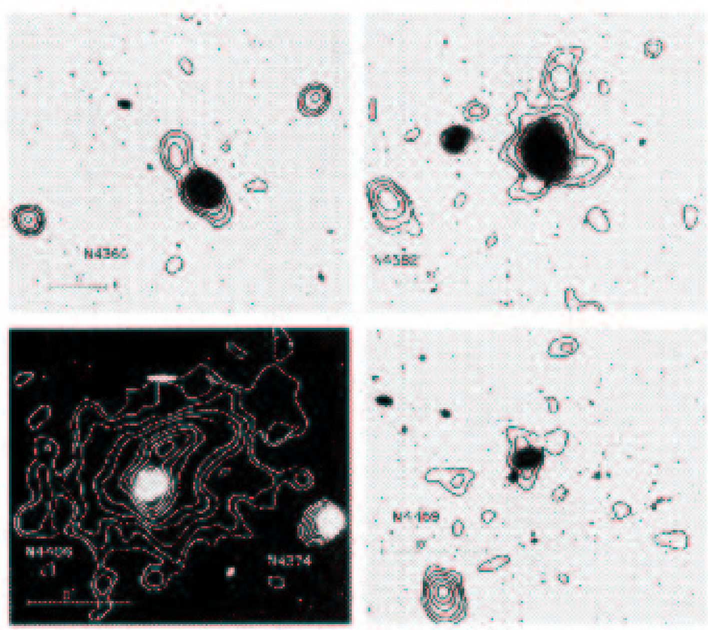

Figure 6: X-rays from ellipticals (Forman & Jones 1985, ApJ, 293, 102)

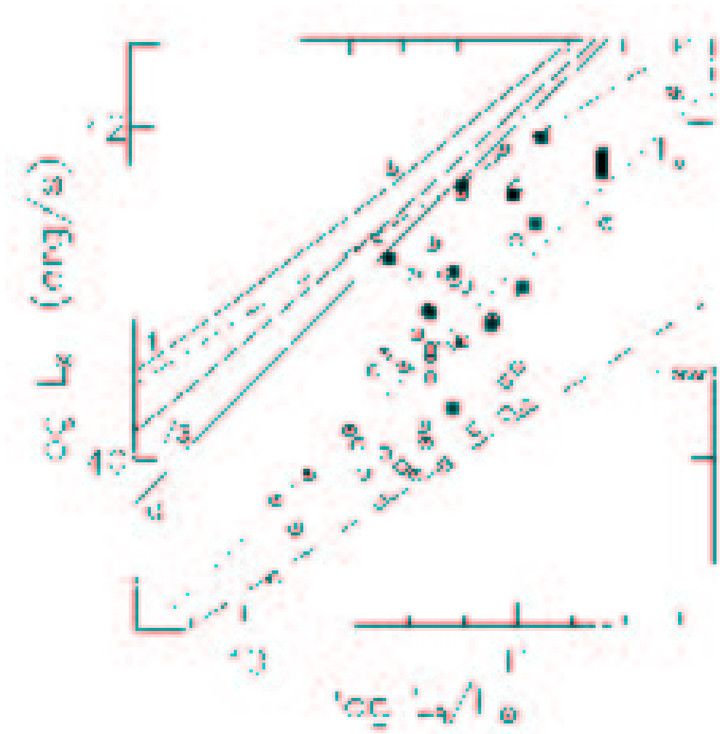

Figure 7: relation between blue and x-ray luminosity (Ciotti et al 1991, ApJ, 376, 380)

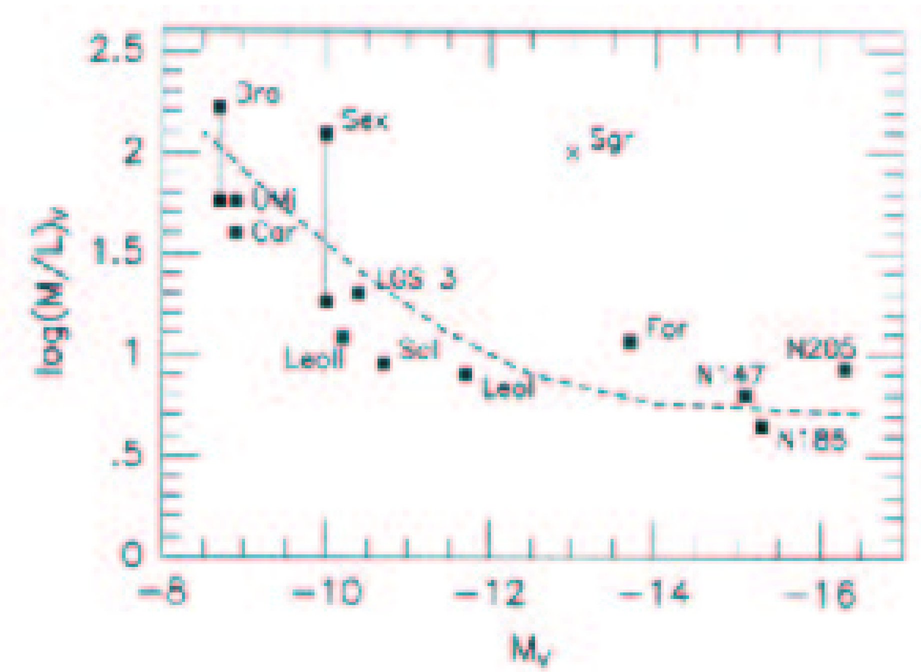

Figure 8: Correlation between M/L_v and M_V for Local Group dwarf spheroidals (Mateo 1997, ASP Conf 116, 266)