Lecture 4 : Spiral galaxies

1 Theories of spiral structure

It has been suggested that the arms do not contain any fixed population of stars but that stars form a **density wave**, a stellar ‘traffic jam’ where stars are crowded together densely.

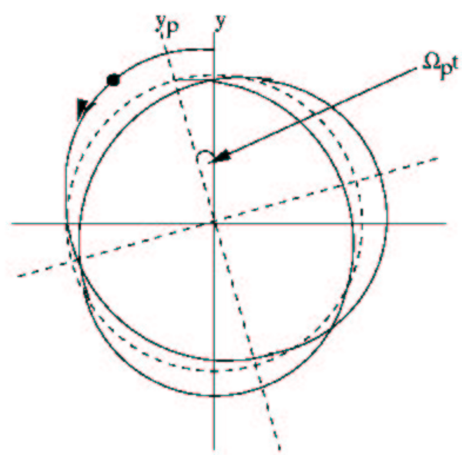

Figure 1: A noninertial coordinate frame (x_p, y_p), rotating within a fixed inertial frame (x,y) with the angular pattern speed $\Omega_p = \Omega - \kappa/2$. The dashed circle corresponds to the circular orbit of the guiding centre, the solid line represents the orbital motion of the star. The orbit is closed in the rotating frame.

Let us first recall that stars in the disc (like in our Milky Way) move in nearly circular orbits. It is therefore useful to have approximate solutions based on perturbed circular orbits. In particular, one has radial simple harmonic oscillations of frequency κ superposed on circular orbits of angular frequency Ω (the **Epicyclic approximation**), so that the radius of the star varies as,

$$r = r_c + x = r_c + X\cos(\kappa t + \psi)\,, \tag{1}$$

where r_c is the radius of the circular orbit and the constant ψ fixes the initial radius and X is the amplitude of the radial motion. The *guiding centre* moves with an angular speed $\Omega(r_c)t$ so that its azimuth $\phi_{gc} = \Omega(r_c)t$. One also has,

$$\kappa = \left[\frac{\partial}{\partial r}(\frac{\partial \phi}{\partial r})\right]_{r_c} + \frac{L_z^2}{r_c^4} = \left[r\frac{d\Omega^2}{dr} + 4\Omega^2\right]_{r_c}. \quad (2)$$

where r_c denotes the circular orbit. In the limit of rigid body rotation, one has $\Omega(r)$ =constant, so that $\kappa = 2\Omega$. for Keplerian motion, one has $\Omega(r) \propto r^{-3/2}$ so that $kappa = \Omega$, which implies closed elliptical orbit. For flat rotation curve one gets $\kappa = \sqrt{2}\Omega$. The range of plausible values of κ is essentially $\Omega \leq \kappa \leq 2\Omega$, which implies that stars oscillate slowly around the circular orbit.

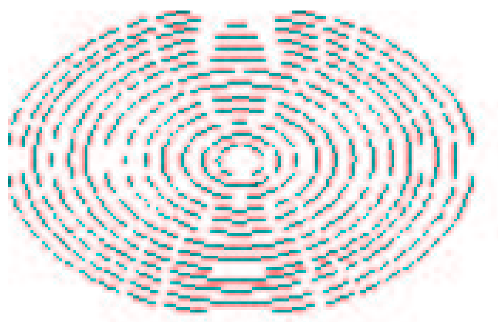

Figure 2: Nested oval orbits—in the rotating frame the orbits are closed.

If we initially place the stars with their guiding centres spread around the circle at r_c and set $\psi = 2\phi_{gc}(0)$ for each, they will initially lie on an oval with its long axis pointing along $\phi = 0$. At time t, the guiding centres will move to $\phi_{gc}(t) = \phi_{gc}(0) + \Omega t$. They stars will advance on their epicycles so that they will now be at radius $r = r_c + x$, with ,

$$x = X\cos(\kappa t + 2[\phi_{gc}(t) - \Omega t]) = X\cos([2\Omega - \kappa] - 2\phi_{gc}(t)). \quad (3)$$

The long axis of the new oval will point in a direction given by,

$$(2\Omega - \kappa)t - 2\phi = 0 \quad , \text{or} \phi = (\Omega - \kappa/2)t \equiv \Omega_p(t), \quad (4)$$

where Ω_p is the *pattern speed*, meaning that the pattern of stars with guiding centre r_c will return to its original state after time $2\pi/\Omega_p$, although the stars themselves would complete their orbits around the centre in time $2\pi\Omega$. For an m-armed spiral we should set $\psi = m\phi_{gc}(0)$ so that the patterns speed is $\Omega_p = \Omega = \kappa/m$.

What does this mean? Suppose we observe the spiral pattern from a rotating frame with speed Ω_p. Now if $\Omega_p = \Omega$, then we simply witness the epicycles

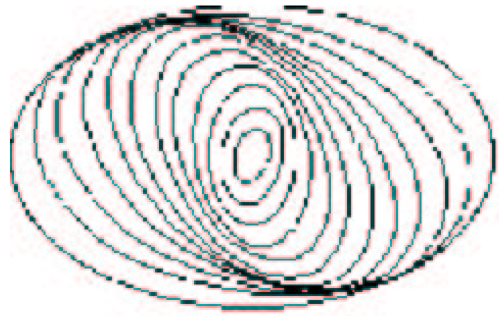

Figure 3: If the orbits are twisted then there will be a fixed spiral pattern in the rotating frame.

with no orbital motion (we are going with the guiding centres). If now $\Omega_p = \Omega - /kappa/2$, then after one epicycle, a star has moved through half an orbit and after second epicycle it is back to the original position, and the orbit is closed in this frame.

Consider now a nested set of orbits with κ varying with r, that is the position angle of ellipses will rotate with r. It is possible to create a two-armed spiral from this set of nested oval orbits. If Ω_p were constant for all r then this spiral pattern would stay fixed in the rotating frame. In general, the pattern speed Ω_p varies with r_c so that this spiral will also wound up in time, but it will longer now because Ω_p is much smaller than Ω, longer by a factor $\Omega/\Omega_p \sim 3$ (for MW). This is a small improvement but is still not adequate. If, however the mutual gravitational attraction of stars and gas at different radii can balance the kinematic spiral's (what we have set up with nested ovals) tendency to wind up, and produce an effective pattern speed, that is almost independent of radius, then the pattern will rotate rigidly with a single pattern speed. This is the premise of the *Quasi-stationary spiral structure (QSSS)* hypothesis of Lin and Shu.

In other words, one first starts with the nested oval orbits, that is orbits which correlate with one another slightly, this leads to density enhancements, which modifies the gravitational field and attracts material to ultimately make the orbits correlate abit more than earlier, gradually strengthening the spiral arms.

Stars encounter the spiral arms with frequency $m(\Omega_p - \Omega)$. One adds a forcing term to the epicyclic equations, to describe the response of the stars to the enhanced potential of the arms (BT section 3.3). The calculations have been done only for tightly wound spirals and one finds that stars respond so as to build up the arms only if the perturbing frequency $m|\Omega_p - \Omega(r)|$ is smaller than the epicyclic frequency $\kappa(r)$ at that radius. There are two radii where there are resonances. There can be a resonance if this frequency coincides with the epicyclic frequency. The *inner Lindblad resonance* is where $\Omega_p = \Omega + \kappa/m$ and the *outer Lindblad resonance* is where $\Omega_p = \Omega - \kappa/m$. Stars beyond the outer resonance, for example, will find the periodic pull of the spiral faster than their epicyclic frequency, and will not be able to respond to strenghthen the spiral. Density waves cannot therefore cross the Lindblad resonances. In fact, for $m \geq 2$, this restricts the density wave to operate on a small annulus of the disc, and therefore one expects two-armed spirals to be common. The density waves get absorbed at the resonances (like ocean waves in a beach).

Figure 4: M51 with its companion.

It is however not clear if QSS density wave theory is correct. Among many worries are the observations that (1) passage through many arms will eventually increase the dispersion which will ultimately choke the process of strenghthening the arms, that (2) many galaxies have other sources of density waves, like tides (Toomre and Toomre 1972). Many galaxies however clear signatures of tidal interactions, and it is possible that tidal interactions help to grow spiral pattern, like the grand design spiral M51, or M100 which show close companions. Bars can also generate spiral patterns (Sanders and Huntley 1976) although they need a lot of dissipation to do so. Numerical simulation seems to show that discs can form bars easily. This *bar instability* is suppressed if there is high dispersion (as for spiral density waves). Recall that about half of all disc galaxies have central bars. Of historical interest is the idea (Ostriker and Peebles 1973) that dark halo may suppress bar instabbilities (dynamics of the inner regions are not much influeneced by the halo). There are also the flocculent spiral, whose spiral arms are weak and fragmentary (almost a third of all spirals are flocculent).

Figure 5: NGC 4414 is an example of a flocculent spiral.

Density wave theory however rightly predicts that (1) dust lanes are usually found on the concave sides of the arms : since the local $\Omega(r)$ exceeds Ω_p, the dust bearing gas is being compressed on this side (in other words, the density waves are overtaken by gas), (2) strong HI emission will be from in front of the optical emission peak which is caused by stars—stars take some time to form and so the peak of the density wave is ahead of the optical peak, and that (3) the arms are usually trailing : it turns out that in a trailing spiral, the inner parts of the disc exert a torque on the outer disc transfering angular momentum outward and allowing material at small radii to move inward; a leading arm would need input of energy from outside.

1.1 Disk instabilities

Disc stars can reinforce a spiral wave only if their random motions are small, so that they do not stray out of the arms. Toomre showed in 1964 that for a rotating disc of stars, axisymmetric waves can grow when the disc is ‘cold’, and the parameter ,

$$Q \equiv \frac{\kappa\sigma_r}{3.36G\Sigma} \leq 1 \tag{5}$$

where σ_r is the velocity dispersion and surface density is Σ. This parameter is called Toomre's Q parameter.

One can crudely argue in the following way. Consider an overdense region of radius R in a non-rotating disc. The collapse time scale is given by R/V where $V \sim \sqrt{GM/R}$ so that $t_{coll} \sim \sqrt{R^3/GM} \sim \sqrt{R/G\Sigma}$. The time scale for stars to wander off from this region is $t_{esc} \sim R/\sigma$. So the condition for stability is given by $t_{esc} > t_{coll}$ implying $R_J \leq \sigma^2/G\Sigma$. Consider now a rotating disc, with the local angular velocity is Oort's constant B. We know that $B = \kappa^2/4\Omega$ locally and also that $\kappa \sim (1-2)\Omega$ so that $B \sim \kappa/3$. This overdense region will be stabel if the centrifugal force is larger than gravity, that is $RB^2 > GM/R^2 = G\Sigma$. The critical size for stability is therefore $R_{rot} > G\Sigma/B^2$.

Combining these two relations, one can argue that the disc is unstable in the range $R_J < R < R_{rot}$ and stable if $R_J > R_{rot}$. This implies that $\sigma\kappa/(3G\Sigma) > 1$. For a rigorous derivation, one sets up a time dependent perturbation in a disc of type $\exp(i[\mathbf{k}\cdot\mathbf{r} - w])$, solves the CBE to find dispersion relations for w, and finds the condition for which w is real, for stability (BT section 6.2).

In the solar neighbourhood, one has $\sigma \sim 30$ km/s, $\Sigma \sim 50 M_\odot$ pc^{-2} and $\kappa \sim 36$ km/s/kpc, so that $Q \sim 1.4$ safely in the stable range.

2 Central black holes

The centres of galaxies (both ellipticals and spirals) have very unusual environments due to deep gravitational potential there. The stellar density, ISM gas density and pressures are all very high. It is also believed that perhaps all galactic nuclei contain massive black holes (BH). One infers the presence of the BH from the velocity field of tracer populations like gas and stars, which defines $d\phi/dr$ and one then finds $M(r)$. It however becomes difficult if there is significant anisotropy in velocity field, especially in bright ellipticals. For these cases, one uses the rotation of gas.

The first attempt was made for M87, assuming isotropic dispersion of stellar velocities—a BH of mass 10^9 $M_\odot$was inferred. Binney & Mamon 1982 however showed that introducing radial anisotropy obviates the need for a central BH. One could independently show that the anisotropy was fairly low though (van der Marel 1994). One has also tried (Richstone & Tremain 1988) to deproject the surface brightness profile $I(r)$ to get the volume brightness $j(r)$, then construct a group of orbits satisfying the constraints of $j(r), V_{rot}$ and σ, and basically build a model of the galactic potential. The best result was from HST image of a rotating gaseous disc, implying a BH of 3×10^9 $M_\odot$.

The HST image of M31 showed a double nucleus. For Milky Way, one now has good radial velocity and proper motion data for hundreds of stars within the central parsec of Sgr A^*. Within this radius, the orbits are isotropic with Keplerian dependences and is consistent with $M(r)$ being constant within a parsec, implying a mass of $2.8 \pm 0.3 \times 10^6$ $M_\odot$. Within 0.01 pc, the orbital timescales are short and tracking the orbits over years the central density appears to be very large ($\geq 2 \times 10^{12}$ $M_\odot$ pc^{-3}), for which a black hole is the most plausible hypothesis (a cluster of neutron stars or white dwarfs will quickly evaporate via two-body relaxation).

It now appears that central BHs are found in all classes of galaxies with bulges. There is also a strong correlation between the black hole mass and the bulge mass, spanning 3 decades in black hole mass. The correlation is poorer with total galaxy luminosity, especially there is no correlation with disc luminosity. This implies that discs have little influence in the formation of BHs. There is an excellent relation with BH mass and the bulge velocity dispersion. Recall that galaxies with high dispersion (for a given bulge luminosity) have smaller effective radius, so they are more compact, and perhaps they collapsed more through dissipation. Perhaps BH mass is linked to the dissipative formation of bulges.

3 Star formation

The strength of a Hα line and the far-infrared luminosity are common indicators of the star formation rate in galaxies. The Hα line strength is simply proportional to the ionizing flux of hot young stars, and therefore to the number of these stars which is a direct measure of the current star formation rate. The far-IR luminosity is from warm dust grains which surround the star formation regions. These grains are heated by UV photons and reradiate in far-IR.

The colours and the equivalent width of Hα line have clues about the star formation history of the galaxy. A high star formation rate in the past produces a number of low mass red stars which make the galaxy redder than average. The equivalent width of Hα line is an indicator of the number of stars that are currently formed relative to the average star formation rate of the history of the galaxy. Comparing this equivalent width with colour one has an idea of the ratio present

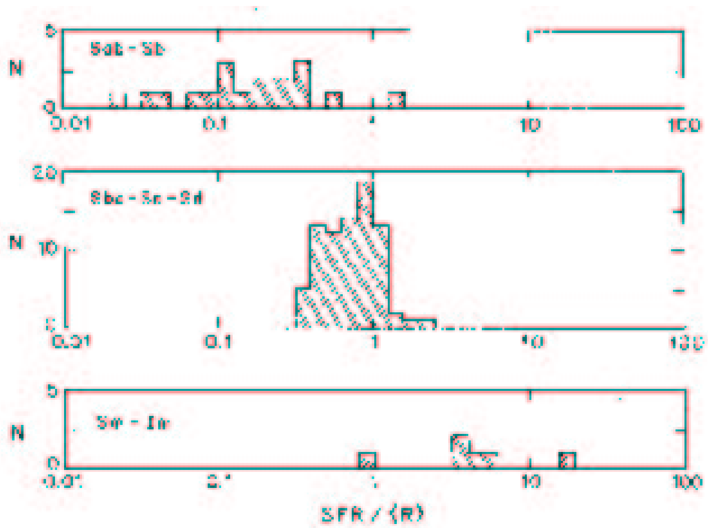

Figure 6: Distribution of current SFR normalized by average past rate (Kennicutt et al 1983, ApJ, 272, 54).

$SFR/ < SFR >$ in galaxies. This ratio is close to unity for Sc galaxies, smaller for Sa and Sb and higher for Sd and later (Sandage 1996).

Kennicut et al (1983, ApJ, 272,54) found the correlation that

$$L_{H\alpha} \sim 10^{41} erg/s (SFR/M_\odot/yr) \qquad (6)$$

It can be understood on the basis of the case B approximation, that photons of the Lyman continuum are completely absorbed and reemitted as Lyman-α or Hα and so on. However its utility is limited by reddening, also its dependence on IMF etc.

There is a strong relation between SFR and surface density spanning many decades

$$\Sigma_{SFR}(M_\odot/yr/kpc^2) \sim 2.5 \times 10^{-7} \Sigma_{gas}^{1.4\pm0.15} (M_\odot pc^{-2}) . \qquad (7)$$

Crudely speaking, one can argue that SFR $\propto$ (gas density)/ (free fall time) $\propto$ $\rho/rho^{-1/2} \propto \rho^{1.5}$. Incidentally, one also obtains by considering the orbital period a relation $\Sigma_{SFR} \sim 0.017 \Sigma_{gas} \Omega_{gas}$, that is, around a tenth of gas is converted to stars per orbit. **Starbursts** are efficient producers of stars partly because higher gas density and shorter orbital time scales.

A very interesting correlation exists between the radio and FIR luminosities of galaxies (the **radio-FIR relation**). The radio emission is from synchrotron radiation of cosmic ray electrons, which are accelerated by SN remnants, whereas the

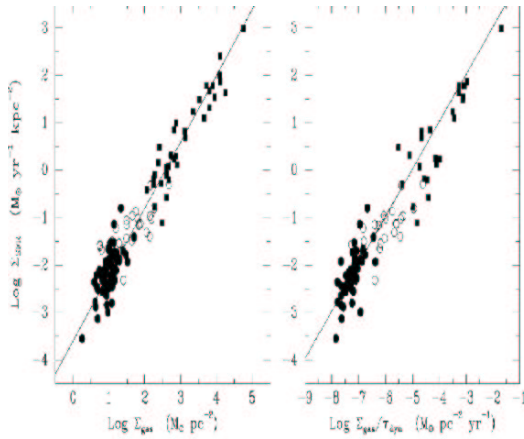

Figure 7: Left: Schmidt law; Right–combined with orbital time (Kormendy 1998)

FIR luminosity is from warm dust in molecular clouds heated by massive young stars. It is expected that there should be correlation since both are connected to star formation. However the lack of scatter in the relation has been surprising and there is no good explanation as yet.

4 Typical spectra

The average spectra of different Hubble types show that SO galaxies mostly emit in long wavelengths, with absorption lines typical of cool K stars. By contrast, Sc galaxies emit most of its light in blue and near UV, it also shows a number of emission lines from gas heated by young stars. The example of a blue starburst shows a number of gas emission lines.

5 Local Group

Our Milky Way belongs to a loose collection of galaxies called the **Local Group**. The most massive members are the Andromeda and the Milky Way, and M33, all being spirals. There are no ellipticals in this group (apart from the atypical M32). Most members are either irregulars (like SMC and LMC) and dwarf spheroidals. There are about 40 known galaxies, but there have been some new discoveries of hidden galaxies behind the Milky Way plane. The luminosity function of the Local Group galaxies follows a Schechter LF form (van den Bergh, 1992, A& A, 264,

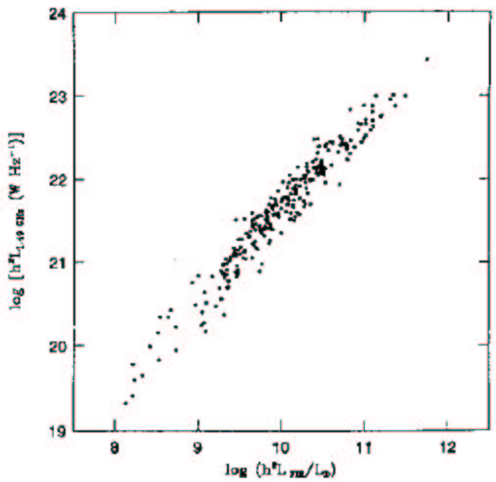

Figure 8: The radio-FIR correlation for strong sources (excluding active galaxies). The radio luminosity is at 1.49 GHz (W/Hz) and the FIR luminosity is at 60μ in solar units (Soifer et al 1989).

75). For details of member galaxies look at http://www.ast.cam.ac.uk/∼mike/ local_members.html

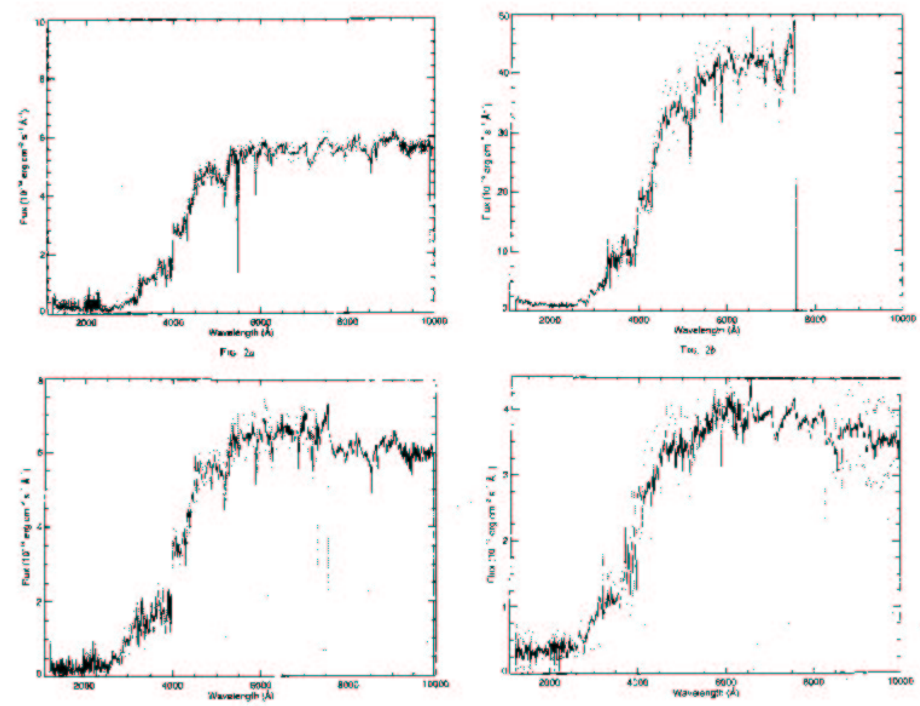

Figure 9: Average spectra of an elliptical, bulge, S0 and Sa galaxy (Kinney etal 1996, ApJ, 467, 38)

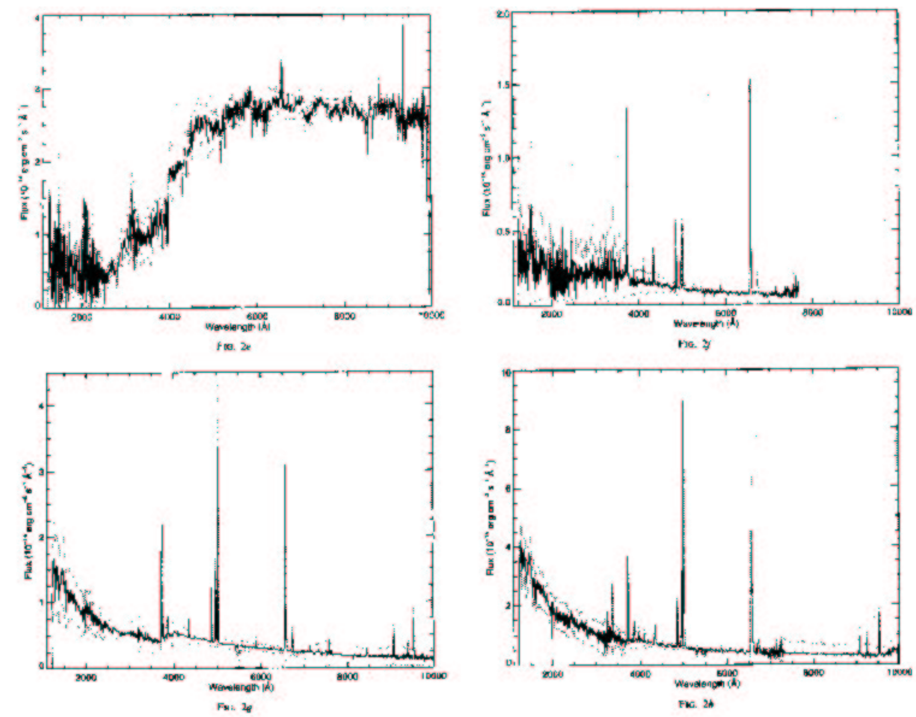

Figure 10: Average spectra of Sb, Sc, and two blue starburst galaxies (Kinney etal 1996, ApJ, 467, 38)

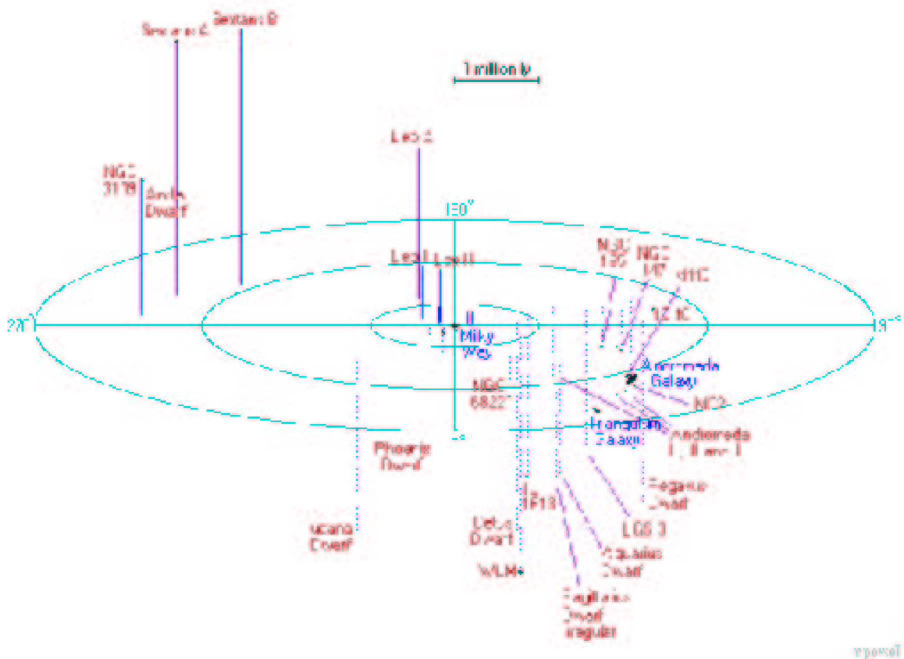

Figure 11: The Local Group.