Lecture 2 : Elliptical galaxies/ Galaxies as stellar systems

Since the Hubble type correlates roughly with other physical parameters, it has been very useful to distinguish galaxies according to the Hubble sequence. But this morphological classification is now subdivided into smaller groups. For example, ellipticals are mainly divided into :

- cD galaxies – These are large and bright objects and found only in the centres of large rich clusters. Their absolute B magnitudes vary between -22 to -25, with masses of order 10^{13} solar masses. The specific frequency of globular clusters $S_N \sim 15$.
- Normal ellipticals – These are objects with high central surface brightness. The blue magnitudes range between -15 to -23 and have masses in the range $10^8 - 10^{12}$ solar masses, with $S_N \sim 1 - 10$.
- Dwarf ellipticals– They are very different from normal ellipticals, with low surface brightness. Absolute B magnitudes vary between -13 to -19 with typical masses in the range $10^7 - 10^9$ solar masses.
- Dwarf spheroidals—These are at the extreme end of teh low luminosity, low surface brightness dwarf ellipticals, and have been found only in the vicinity of the Milky Way.
- Blue compact dwarf galaxies–They are small galaxies with unusual blue colour, perhaps galaxies with vigorous star formation rate. Blue magnitudes vary between -14 to -17 with masses of order 10^9 solar.

Many old ideas about ellipticals have changed recently. It used to be thought that ellipticals (a) were devoid of gas and dust, (b) were made of a single old population of stars and (c) were dynamically relaxed systems. All these ideas have changed now. Let us first look at the important observed properties of elliptical galaxies.

Since galaxies look projected onto the plane of sky, many observed parameters describe projected quantities. For example the surface brightness profile $I(R)$ where R is the projected radius is actually the projection of the volume emissivity (with $z^2 + R^2 = r^2$ and $dz = rdr/\sqrt{r^2 - R^2}$),

$$I(R) = \int_{-\infty}^{\infty} j(r)dz = 2\int_R^{\infty} \frac{j(r)rdr}{\sqrt{r^2 - R^2}}, \tag{1}$$

which can be deprojected with the help of,

$$j(r) = \frac{-1}{\pi} \int_r^\infty \frac{dI}{dR} \frac{dR}{\sqrt{R^2 - r^2}} \tag{2}$$

Often this has to be done numerically with iterative procedures.

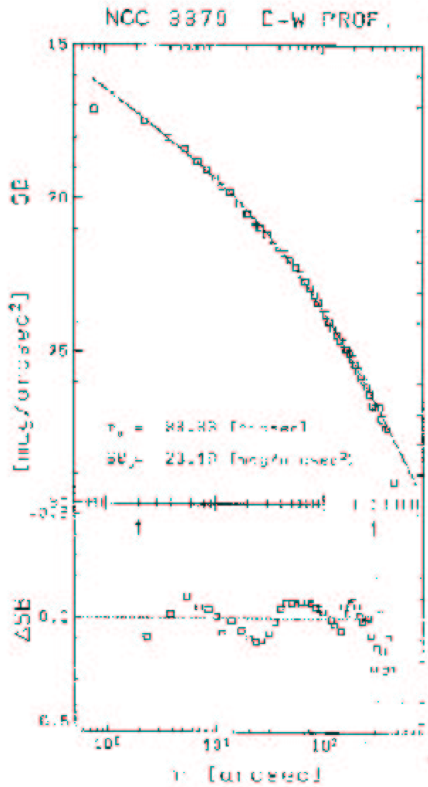

Figure 1: The Jaffe profile fits the surface brightness profiles of ellipticals well (for NGC 3379, from Surma et al 1992, A&A).

A number of analytic fits have been suggested to fit the $I(R)$ of elliptical galaxies. Here are a few important fits:

- de Vaucouleurs law : $I(R) = I_e \exp\left(-7.67\left[(R/R_e)^{1/4} - 1\right]\right)$, where R_e is the radius that contains half the projected light, and I_e is the surface brightness at R_e. The asymptotic behaviours are $I(R) \propto R^{-0.8}$ at small R and $\propto R^{-1.7}$ at large R. Binney (1982) showed that this could be caused by reasonable stellar distribution function.

- Hubble law : $I(R) = I(0)/(1+R/r_0)^2$; since $I(R) \propto R^{-2}$ the total brightness diverges.
- Modified Hubble Law (Isothermal/King profiles) : $I(R) = I(0)/[1+(R/r_0)^2]$ The interesting point about this profile is that the deprojected volume profile is given by, $j(r) \propto 1/(r_0[1+(r/r_0)^2]^{3/2})$ which is theoretically well grounded (see below).

1 Theoretical interlude I : Jeans theorem and applications

You would recall from the discussions on stellar dynamics (First semester) that *stars do not experience significant encounters*. This is because the two-body relaxation time for N objects in a system of radius R and typical velocity v is

$$\tau_{relax} \frac{N}{8 \ln N} \frac{R}{v} \tag{3}$$

Let us recapitulate briefly its derviation. For a star of mass m passing by another with an impact parameter b and velocity v, one has the relevant component of gravitational force, $F_\perp = \frac{Gm^2 b}{((b^2+x^2)^{3/2}}$. Setting the zero time such that $x = vt$, one has the impulse,

$$\delta v_\perp = \int_{-\infty}^{\infty} F_\perp/m dt = \int \frac{Gm}{b^2}(1+(vt/b)^2)^{-3/2} dt = \frac{Gm}{bv} \int (1+s^2)^{-3/2} ds = \frac{Gm}{b^2}\frac{2b}{v}, \tag{4}$$

which is basically acceleration times the passage time. The probability that a star will pass another star in a distance interval $b, b+db$ is $2\pi b db/(\pi R^2$, where R is the system size. For a system with N stars the total number of interactions is $\delta n_b = (2N/R^2) b db$, changing the speed with every interactions. The mean square deflection is $< \delta v_\perp^2 >= (2Gm/bv)^2 \delta n_b$. Integrating this over all b, one gets, $8N(Gm/vR)^2 \ln b_{max}/b_{min}$. We could choose $b_{max} \sim R$ and $b_{min} \sim Gm/v^2$ which is the limit of small deflections, $\delta v_\perp \sim v$. This limit actually means, (since $GM/v^2 = R/N$, as virial theorem says $2(Nmv^2/2) = G(Nm)^2/R$) $b_{min} \sim R/N$. Finaly one has,

$$\frac{< \delta v_\perp^2 >}{v^2} = \frac{8 \ln N}{N}. \tag{5}$$

A star will have to cross the system $N_{relax} = N/(8 \ln N)$ times for relaxation, for $< \delta v_\perp^2 > \sim v^2$, which would take a time $N_{relax} t_{cross}$, which gives the above mentioned timescale by using $t_{cross} = R/v$. For an elliptical galaxy, with $N = 10^{12}$, $R = 10$ kpc and $v = 600$ km/s, the time scale for relaxation is about 10^{17} yr. So two-body relaxation is insignificant. So that the phase space distribution function obeys the **collisionless Boltzmann equation (CBE)**.

Since it is closely related, let us also discuss **dynamical friction** which we will use later. The two-body interactions essentially provide a drag force on a moving object. We have found that the impulse given to a object of mass M moving with speed v by the interactions with a star of mass m is $2Gm/bv$. The star also receives an equal and opposite momentum. The total kinetic energy in the perpendicular direction is

$$\Delta KE_{\perp} = (M/2)(2Gm/bv)^2 + (m/2)(2GM/bv)^2 = 2G^2mM9m+M)/(b^2v^2)\,. \quad (6)$$

This energy basically comes from retardation in the forward motion, which changes by an amount $v_{||}$. At large distances the potential energy is small, so that we can equate the KE :$(M/2)v^2 = \Delta KE_{\perp} + (M/2)(v+v_{||})^2$ giving (neglecting terms with $v_{\perp}^2$), $-\Delta v_{\perp} \approx \Delta KE/(Mv) = 2G^2m(M=m)/(b^2v^3)$. The drag force is simply $F_{drag} = -m\Delta v_{||}$. Integrating over all impact parameters ($2\pi bdb$) and the encounter rate nV one has,

$$\begin{aligned} F_{drag} &= -\frac{4\pi G^2M^2nm\ln\Lambda}{v^2} = -\frac{4\pi G^2M^2\rho\ln\Lambda}{v^2} \\ &= -\frac{4\pi G^2M^2m\ln\Lambda}{v^2}\int_0^V 4\pi v^2 f(v)dv \\ &\quad -\frac{4\pi G^2M^2nm\ln\Lambda}{v^2}\left[\mathrm{erf}(X) - \frac{2X}{\sqrt{\pi}}\exp(-X^2)\right] \end{aligned} \quad (7)$$

where the last equality is for a (isotropic) field star velocity distribution with Maxwellian $f(v)$ with dispersion σ, where $X = v/sqrt\sigma$.

You would also recall the Jeans theorem which says that *any integral of motion is a solution of the time-independent CBE*. It is easily proved as (suppose $I[\mathbf{r}(t), \mathbf{v}(t)]$ is an integral of motion),

$$0 = \frac{dI}{dt} = \frac{d\mathbf{r}}{dt}\cdot\frac{dI}{d\mathbf{r}} + \frac{d\mathbf{v}}{dt}\cdot\frac{dI}{d\mathbf{v}} = \mathbf{v}\cdot\frac{dI}{d\mathbf{r}} - \nabla\Phi\cdot\frac{dI}{d\mathbf{v}}\,, \quad (8)$$

where we have substitued the equations of motions in the last equality (ϕ being the potential). You would recall that the CBE is given by (for a distribution function $f(\mathbf{r},\mathbf{v},t)$),

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f - \nabla\phi\cdot\partial f\partial\mathbf{v} = 0\,. \quad (9)$$

In fact, any function of **r** and **v** that depends only through one or more integrals of motion is *also* a solution of the time-independent CBE.

In the simplest case of an isotropic distribution function, it depends only on the specific energy, that is $f(\mathbf{r},\mathbf{v}) = f(E)$. In a self-consistent system, the gravitational field is related to the mass density by Poisson equation,

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{d\phi}{dr}) = 4\pi G\rho(r) \tag{10}$$

. Suppose we use a boundary condition that $\phi \to 0$ as $r \to infty$ then the escape energy is zero and all stars at radius r have energies between $\phi(r)$ and zero. The mass density $\rho(r)$ is basically the integral of $f(\mathbf{r},\mathbf{v})$ over all velocities, so that,

$$\rho = 4\pi\int_0^{v_e} v^2 f(E(r,v))dv\,, \tag{11}$$

where $v_e - \sqrt{-2\phi(r)}$ is the escape velocity at radius r. Given any function form of $f(E)$ (so that it is positive for all $E < 0$) we can use these two equations to derive the mass density. The **Plummer model** is the simplest, for which the distribution function has the form $f(E) = F(-E)^{7/2}$ for $E < 0$ and $f(E) = 0$ for $E > 0$, where F is a constant. Using equation **??** one has,

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{d(\phi)}{dr}) = K(-\phi)^5\,, \tag{12}$$

where K is a constant. The solution of this ODE is $\phi \propto \sqrt{r^2+a^2}$, which implies a mass density profile $\rho \propto (r^2+a^2)^{-5/2}$. Upon normalizing one gets,

$$\rho(r) = \frac{3Mz^1}{4\pi}(r^2+a^2)^{-5/2}\,. \tag{13}$$

One can then find the corresponding projected mass (or luminosity, for constant M/L ratio) surface density profile. One can show that an **isothermal** distribution, with $f(E) \propto \exp(-E/\sigma^2)$, that is where velocities have a maxwellian distribution (corresponding to a single 'temperature'), Substituting $\phi - 1/2v^2 = E$ and integrating over velocities, one gets

$$rho \propto \exp(\phi/\sigma^2)\,, \tag{14}$$

Putting this in the Poission's equation, one gets,

$$\frac{d}{dr}(r^2\frac{d\ln\rho}{dr}) = -\frac{4\pi G}{\sigma^2}r^2\rho\,, \tag{15}$$

which turns out to be the hydrostatic equation for an isothermal gas, with $\sigma^2 = kT/m$. For the boundary condition $\phi(0) = \infty$, one gets $\rho(r) = \sigma^2/(2\pi G r^2)$ (BT pp.226-228). This is the **singular isothermal model**. For a boundary condition $\rho + (0) = \rho_0$ and $(d\rho/dr)_{r=0} = 0$ (flat density profile at centre) one gets a density profile as in Fig 4.7 of BT. One then finds a flat denisty profile within a radius $r_0 = 3\sigma/(4\pi G\rho_0)^{1/2}$, which is called the *core radius*.

The **King models** assume a truncated isothermal distribution function,

$$f(E) \propto [\exp(-(E - E_0)/\sigma^2) - 1], \tag{16}$$

for $E < E_0$ and zero for $E > E_0$. Integrating and putting in the Poission's equation, one gets

$$\frac{d}{dr}(r^2 \frac{d\phi}{dr}) = -4\pi G\rho_1 r^2 \left[e^{\phi/\sigma_0^2} \mathrm{erf}(\sqrt{\phi/\sigma_0}) - \sqrt{4\phi/\pi\sigma_0^2}(1 + \frac{2\phi}{3\sigma_0^2}) \right]. \tag{17}$$

One can solve this numerically, with boundary conditions at $r = 0$, with $\phi(0) = q\sigma_0^2$ (large q meaning a deep potential) and $d\phi/dr = 0$. The inner region resembles an isothermal sphere with a core radius r_0. The term '$-1'$ in the distribution function tends to reduce the number of stars with high kinetic energy in the outer regions. This makes the density drop abruptly after a certain *truncation radius*,r_t. One defines a concentration parameter as $c = \log(r_t/r_0)$. (BT fgi 4.10) One can find a sequence of King models by varying c (or, equivalently q). It turns out that $c \geq 2.2$ ($q \geq 10$) fit some ellipticals well (and $c = 0.75 - 1.75$ ($q = 3 - 7$) fit globular clusters well). The limit $c \to \infty$ takes one back to the isothermal sphere.

A very useful mass density profile is that of **Jaffe model**,

$$\rho(r) = \frac{Ma}{4\pi} r^{-2} \frac{1}{(r + a)^2}, \tag{18}$$

with a corresponding luminosity density profile. The associated potential is $\phi(r) = GM(1/a)\ln[r/(r+a)]$. The corresponding projected surface brightness profile fits the data very well.

Variation of brightness profiles : Ellipticals of different luminosities have different slopes : lower luminosity Es have steeper slopes, steeper than $r^{1/4}$ law. Dwarf ellipticals are a different story. Compact dEs like M32 have a $r^{1/4}$ profile whereas the diffuse dEs have exponential light profiles (although they are not discs) –perhaps they are a distinct populations and not the low luminosity tail of bright ellipticals.

2 Correlations

Colour-Magnitude relation: Brighter ellipticals appear to be slightly redder.

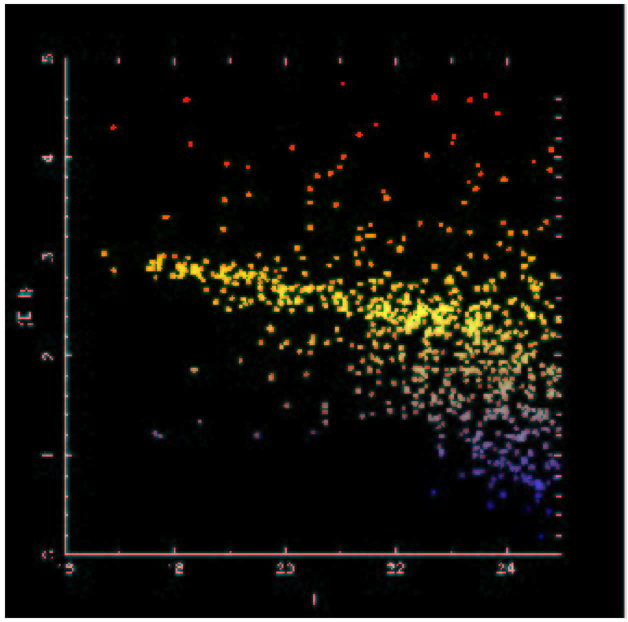

Figure 2: Colour magnitude (B-I vs I) diagram for an intermediate redshift cluster (Abell 2218).

Metallicity-velocity relation: there is a strong correlation between galaxies with deeper potentials (since $\sigma^2 \propto GM/r_e$ with the strength of Mg_2 line. Perhaps this means that for deeper potential, a larger fraction of the gas can form stars (galactic wind is held off for longer), producing more metals.
Size/Luminosity vs surface brightness (Kormendy) relation : Larger galaxies appear to have lower surface brightness. Kormendy found that $R_e \propto I_e^{-0.8}$, and $L_{tot} \propto I_e^{2/3}$. Larger and brighter ellipticals are therefore *puffed up* and have lower densities.
Faber-Jackson relation Faber and Jackson (1976) found that brighter ellipticals have deeper potentials, $L \propto \sigma^n$ with $3 < n < 5$. One could argue for this relation very crudely if ellipticals had constant M/L ratio and surface brightness. Since $\sigma^4 \propto (GM/R)^2 \propto (M/L)^2(L/R^2)L$.
Fundamental plane

The above relations have large scatter (more than measurement errors). It has

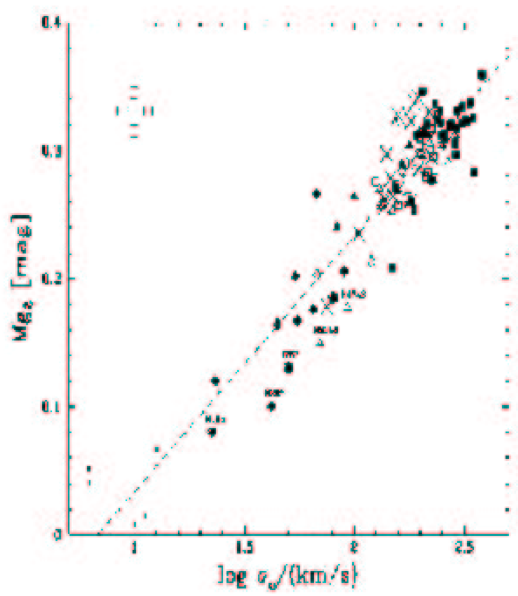

Figure 3: Mg_2 vs. velocity dispersion relation for ellipticals (square) SO bulges (crosses) and dEs (diamonds and small squares) (Bender IAU symp 149).

been suggested that actually there is a good 3-parameter relation and the above relations are mere projections, thereby increasing the scatter. Djorgovsky and Davis (1987) suggested a relation between $\log R_e$ (kpc), $\langle\Sigma\rangle$ (B magnitude/square st) and $\log\sigma_e$ (km/s) : $\log R_e = 0.36\langle\Sigma\rangle + 1.4\log\sigma_e +$ constant. This relation has scatter less than 15%.

One can again very crudely argue for this relation from virial theorem. Virial equilibrium implies that (KE being proportional to PE),

$$\frac{M}{r_e} = c\sigma_0^2\,, \tag{19}$$

where c is a 'structure' parameter that hideas all our ignorance of the details of the structure of the halo. We also have as the definition of the mean surface brightness profile (with half the total luminosity within the effective radius),

$$I_e = \frac{L/2}{\pi r_e^2}\,. \tag{20}$$

These two equations imply,

$$r_e = (c/2\pi)(M/L)^{-1}\sigma_0^2\Sigma_e^{-1}\,, \tag{21}$$

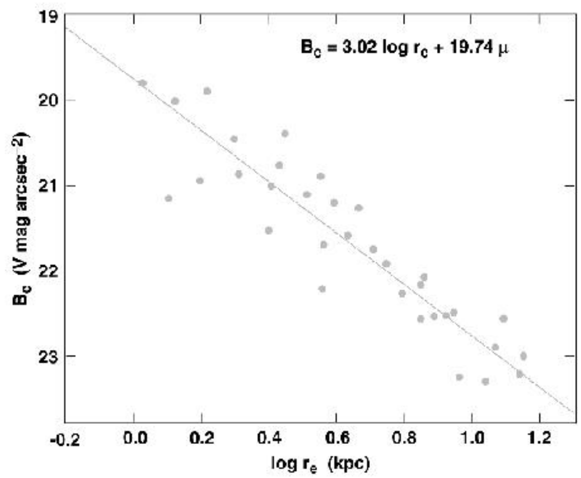

Figure 4: Kormendy relation between effective radius and surface brightness.

implying (assuming constant values of c and M/L) $\log r_e = const + 2\log\sigma_0 - \log < I_e >= const + 2\log\sigma_0 + 0.4 < \Sigma_e >$ (as $< \Sigma_e >= -2.5\log < I_e >$), which is quite close to the observed relation. To make them agree well, one needs the M/L ratio to vary, as $M^{0.2} \propto L^{0.25}$. This means that ellipticals are structurally very similar with very weakly varying M/L ratio.

It appears that these indices do not depend on the environment. Whether or not they depend on the redshift is still uncertain.

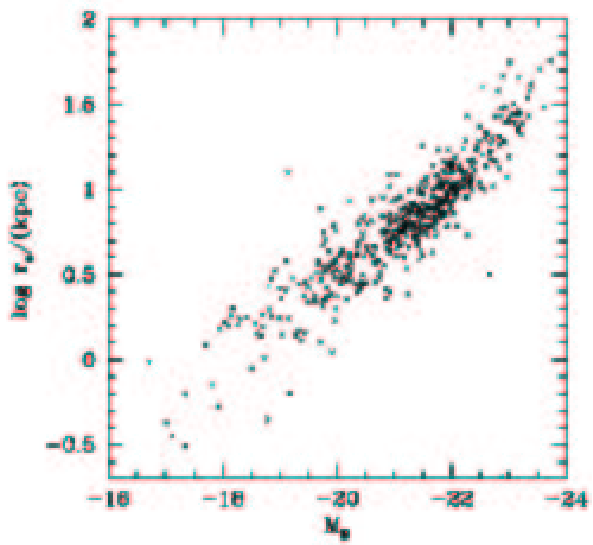

Figure 5: Faber-Jackson relation.

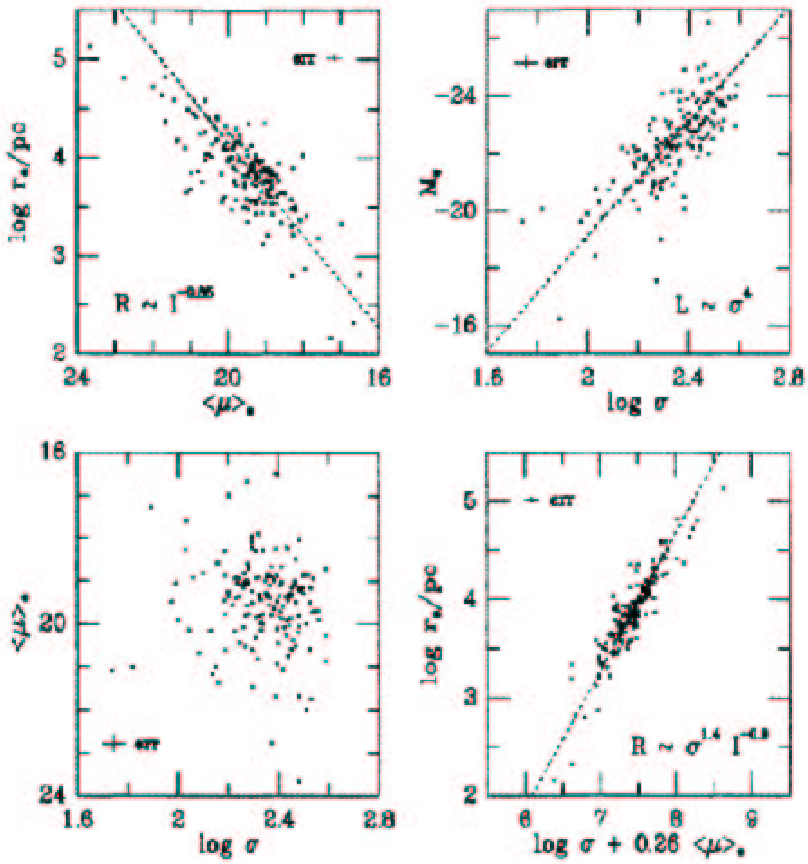

Figure 6: Fundamental plane.