Lecture 6 : Chemical evolution

1 Evidence of chemical evolution

The best evidence for ongoing chemical evolution of our Galaxy comes from the observations of short lived radioactive elements, like ^{26}Al, which has a half life of $\sim 7\times10^5$ yr. The recent COMPTEL measurements of the 1.8 MeV line from this element clearly shows that it has been generated very recently. Another evidence is that the ratio of $^{13}C/^{12}C$ is much larger (~ 0.1) in carbon stars than in the ISM (~ 0.01). This means that recent processing of material is going on in these stars. Over time this material will be dispersed in the ISM and will be incorporated into successive generations of stars.

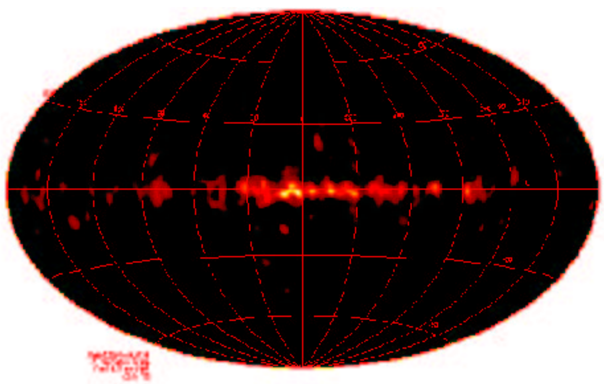

Figure 1: Skymap of ^{26}Al made with COMPTEL. This radioactive element has a half life of about million years, so these radioactive clouds must have been created recently, suggesting recent enrichment.

2 Ingredients

The first ingredient of our discussion on chemical evolution is the **initial mass function (IMF)** and the **star formation rate (SFR)**. The number of stars formed in the mass interval $(m, m+dm)$ in the time interval $(t, t+dt)$ is the stellar birth function, $B(m,t)dmdt$. To simplify matters, we assume that the variables can be separated into two functions,

$$B(m,t)dmdt = \phi(m)\psi(t)dmdt\,, \tag{1}$$

where $\psi(t)$ is the total mass of stars formed per unit time (the SFR), and $\phi(m)$ is called the IMF (normalized so that $\int_{m_L}^{m_U} m\phi)m)dm = 1$. This assumption basically means that IMF is independent of time. One often approximates the IMF as a power law in the range of relevant mases. The ‘industry standard’ IMF is the *Salpeter mass function* (Salpeter 1955),

$$\phi(m) \propto m^{-(1+x)}\,, \tag{2}$$

where $x = 1.35$. This is not strictly true, especially below ~ 1 M $_{\odot}$ (Scalo 1986) but this form is easily handled and has been very useful in interpreting many observations.

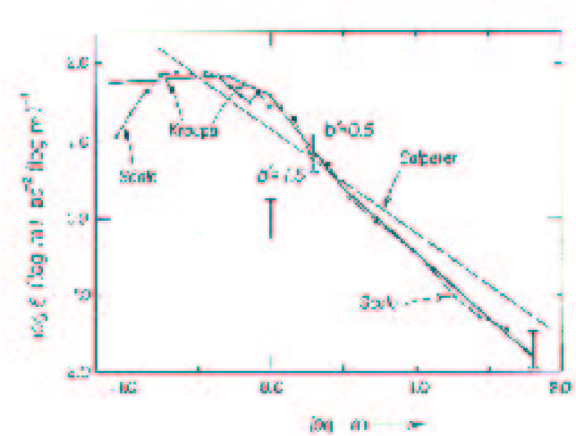

Figure 2: Local IMF from Scalo (1986) (thin line) and Kroupa (1991) (thick line) comapred with Salpeter’s power law formulation (broken line).

The star formation rate has been traditionally modelled, following Schmidt (1959), as being proportional to some power of the gas density,

$$\psi \propto \rho^n \tag{3}$$

where ρ is the local gas density. The original observations of Schmidt had indicated $n \sim 2$ but he had used observed HI mass density whereas the main source of star formation, the molecular clouds, were not used. These parameterizations are very tricky, but this is the conventional way.

One can then write the first equation of chemical evolution (CE), (with g is the mass in gas phase and s is mass in stars at any given time), $M = g + s$, describing the total mass of the system. One can also write the conservation of mass as,

$$\frac{dM}{dt} = \frac{dg}{dt} + \frac{ds}{dt} = F - E \tag{4}$$

where F is the rate of mass flow into the galaxy (say, from infall) and E is the rate of mass flow out of the galaxy (say, due to a galactic wind). The fraction of mass in gas is written as $\mu = g/M$. The stellar mass obeys the equation,

$$\frac{ds}{dt} = \psi - e\,, \tag{5}$$

where e is the ejection rate of material from stars. The gas mass similarly follows,

$$\frac{dg}{dt} = F - E + e - \psi\,. \tag{6}$$

Suppose metals are ejected from stars at the end of their lifetime (τ_m for stellar mass m), leaving a remnant of mass m_{rem} then the ejection rate is given by,

$$e(t) = \int_{m_{\tau=t}}^{m_U} (m - m_{rem})\psi(t - \tau_m)\phi(m)dm\,. \tag{7}$$

This approximation is not valid for stars hotter than B since they lose mass on the MS branch, so this is invalid for $t \ll 10^7$ yr. The abundance of a stable (not radioactive) element in the gas is given by,

$$\frac{d}{dt}(gZ) = e_Z - Z\psi + Z_F F - Z_E E\,, \tag{8}$$

where the first term denotes the total amount of that element ejected by stars (both newly synthesized and old, pre-birth material), the second term denotes the incorporation of metals into new stars, the third and fourth are due to replenishment and loss due to inflow and outflow of gas.

There are three useful quantities which do not depend on the details of SFR $\psi(t)$. The **returned fraction** R is defined as,

$$R \equiv \int_{m_\tau}^{m_U} (m - m_{rem})\phi(m)dm, \tag{9}$$

where m_τ is the current turn-off mass, usually assumed to be ~ 1 M$_\odot$. This is the fraction of mass put into stars at a given time which is returned to ISM later on, in the lifetime of a solar mass star. One also defines a **lock-up fraction**, α, which is the fraction of mass locked up in long-lived stars and remnants. Obviously,

$$\alpha \equiv 1 - R\,. \tag{10}$$

For $m_\tau \sim 1$ M$_\odot$, one has $\alpha \sim 0.7 - 0.8$ depending on the IMF. Finally, there is the **yield**, p, which is the mass of new metals ejected per unit mass that is locked into stars. If $q_Z(m)$ is the mass fraction of a star (with mass m) that is converted into metals and ejected, then the yield is,

$$p = \alpha^{-1} \int_{m_\tau}^{m_U} m q_Z(m) \phi(m) dm\,. \tag{11}$$

If stars of mass M formed in a single burst a long time ago, then the mass of new metals ejected is $q_z = \alpha p$. One can find the values of mq_Z from the work of Woosley & Weaver (1995).

2.1 Instantaneous recycling approximation

To make things analytically tractable, it is helpful to make the assumption that all processes involving stellar evolution and reclying of metals have timescales much shorter than that of galactic evolution. This is called the *instantaneous recycling approximation*. It allows one divide the stellar population into a part that (1) live forever ($m < m_1$) and another that (2) die instantly ($m > m_1$). The lower mass limits in the integrals above can then be taken to be m_1 that is time-independent, and the term $(t - \tau_m) = t$. One then writes the ejection rate $e = R\psi(t)$ and the ejection rate of metals (new plus old), as (with t as the current time),

$$e_Z(t) = RZ(t)\psi(t) + p\alpha(1 - Z(t))\psi(t) \sim RZ(t)\psi(t) + p\alpha\psi(t)\,, \tag{12}$$

the second equality follows since $Z \ll 1$. We then find,

$$\begin{aligned} \frac{ds}{dt} &= \alpha\psi\,, \\ \frac{dg}{dt} &= -\alpha\psi + F - E \\ \frac{d}{dt}(gZ) &= -\alpha Z\psi + p\alpha\psi + Z_F F - Z_E E\,, \end{aligned} \tag{13}$$

implying (since $gdZ/dt = d(gZ)/dt - Zdg/dt$), $g\frac{dZ}{dt} = p\alpha\psi + (Z_F - Z)F - (Z_E - Z)E$.

Let us consider the mean metallicity of all stars. If we now assume the IMF to be constant, then the total mass of stars at t is, $s(t) = \alpha\int_0^t \psi(t')dt' \equiv \alpha\bar{\psi}t$. The total mass of new metals ever ejected is obtained by summing over time for the ejection rate of new metals, $\int_0^t\int_m mq_Z(m)\psi(t')\phi(m)dt'dm = \int_0^t \psi(t')dt'\int_m mq_Z(m)\phi(m)dm = \frac{s}{\alpha}\times p\alpha = ps$. But this is also equal to the sum of mass sZ_s of metals in stars and gZ in gas. We therefore have (since $sZ_s + gZ = [(1-\mu)Z_s + \mu Z]M = ps = (1-\mu)pM$),

$$Z_s = p - \frac{\mu}{1-\mu}Z\,, \tag{14}$$

which shows that as $\mu \to 0$ (that is gas is exhausted), $Z_s \to p$, that is, all the metals ever made and ejected (ps) are incorporated into later generation stars. The yield is then just the mean metallicity of stars, so that $p \sim 0.8Z_\odot$, or simply, $p \sim Z_\odot$.

The instantaneous approximation is certainly inadequate for elements like Fe which is mostly produced by SN I and which takes a long time. There have been some recent improvements, like the suggestion of Pagel (1989) of a ‘delayed production approximation’.

2.2 Closed box model

Let us now look at some extreme regimes for illustrative purpose. In the *closed box* model, one assumes that (1) the system is isolated (M is constant), (2) the system is well mixed at all times ($Z = Z(t)$ is the abundance in the gas <u>and</u> the newly formed stars), (3) the system is initially pristine ($g(o) = M, Z(0) = s(0) = Z_s(0) = 0$), and (4) the IMF and yields are constant. Since $F = E = 0$, we get from equations 13 and 2.1, $g\frac{dZ}{dg} = -p$, which gives,

$$Z = p\ln g(0)/g = p\ln M/g = p\ln\mu^{-1}\,, \tag{15}$$

which is valid for $Z \ll 1$. This is the <u>most important</u> result of the closed box model.

2.3 The ‘G-dwarf problem’

Let us consider an application of these ideas. It appears that there are too few metal poor G dwarfs (also K and M dwarfs) stars in the solar neighbourhood compared to the simple closed-box model prediction. Consider the cumulative

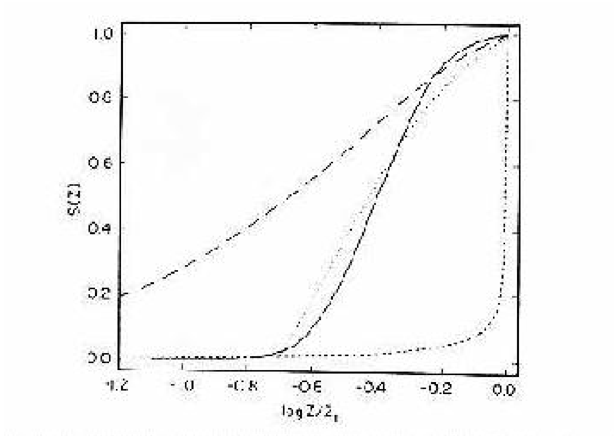

Figure 3: Tinsley's plot for the G-dwarf problem. Solid line is data, long dashed line is from closed box model and the short dashed line is from an extreme infall model. The dotted line assumes a finite initial metallicity (see below).

metallicity distribution of stars ever formed. Then all the stars that formed when the gas fraction was $\geq \mu$ is (with the subscript 1 representing the current epoch)

$$\frac{s}{s_1} = \frac{1-\mu}{1-\mu_1}. \tag{16}$$

These stars formed when the metallicity of the gas was less than $p \ln \mu^{-1}$. The fraction of stars with metallicities less than Z, $s(Z)$ is then given by,

$$s(Z) = \frac{s}{s_1} = \frac{1-\exp(-Z/p)}{1-\mu_1} = \frac{1-\mu_1^{Z/Z_1}}{1-\mu_1}. \tag{17}$$

eliminating the yield by using $p = Z_1/(\ln \mu_1^{-1})$. Tinsley compared this prediction (long-dashed line in figure) with observations (solid line) in 1980 (things haven't changed much since then; in fact new data (Jorgensen 2000) show that the problem may be worse than shown here), which clearly shows the paucity of metal poor G-dwarfs compared to the closed box prediction.

The suggested remedies for this problems are:

- The IMF is not constant and there were more massive stars earlier – contrived?
- The gas is pre-enriched by a previous generation of stars. In the Eggen-Lynden-Bell-Sandage scenario, massive halo stars would pollute the disk

material (Ostriker & Thuan 1975) before it collapses to form stars. One then adds an initial metal abundance Z_0 to the equations and find $Z + Z_0 + p \ln \mu^{-1}$ and $s(Z) = [1 - \mu_1^{(Z-Z_0)/(Z_1-Z_0)}]/(1-\mu_1)$ which is the dotted line in the figure.

- Infall increases metallicity. Pagel (1997) describes several such models (the short-dashed curve in the figure is an extreme case)