Stellar systems: two-body relaxation

Collection of a large number of stars, held by the combined gravitational forces are called stellar systems. If the stellar density is large, two-body gravitational relaxation is important in the system: these are called "collisional" stellar systems. In more dilute systems the relaxation could take longer than the age of the universe; these are called "collisionless" stellar systems.

Let a stellar system consist of N stars of average mass m per star, and let R be the size of the system. The number density of stars is $n \sim N/R^3$. The total mass of the system is $M = Nm$. The two-body relaxation time in the system can be estimated in the following manner. The change in the velocity v of a star in one encounter is

$$(\delta v_\perp)_1 \sim \frac{Gm}{b^2}\frac{2b}{v} \sim \frac{2Gm}{bv}$$

where b is the impact parameter in the encounter. In multiple encounters this adds randomly, so $(\delta v_\perp)^2$ grows with time. The rate of increase of the mean square speed is then

$$\begin{aligned}
\frac{d}{dt}(\delta \bar{v}_\perp)^2 &\sim \int_{b_{\min}}^{b_{\max}} (\delta v_\perp)_1^2 \, nv \, 2\pi b db \\
&\sim \frac{8\pi G^2 m^2 n}{v} \int_{b_{\min}}^{b_{\max}} \frac{db}{b} \\
&\sim \frac{8\pi G^2 m^2 n}{v} \ln\left(\frac{b_{\min}}{b_{\max}}\right)
\end{aligned}$$

The relaxation time is then

$$t_R = \frac{v^2}{\frac{d}{dt}(\delta \bar{v}_\perp)^2} = \frac{v^3}{8\pi G^2 m^2 n \ln(b_{\max}/b_{\min})}$$

Noting that $v^2 \sim GM/R \sim GmN/R \sim GmnR^2$, defining crossing time

$$t_{\rm cr} = R/v = 1/\sqrt{G\rho} = 1/\sqrt{Gmn}$$

and identifying

$$b_{\max} \sim R \sim (N/n)^{1/3},$$
$$b_{\min} = \text{inter-star distance} \sim n^{-1/3}$$

we have

$$\frac{b_{\max}}{b_{\min}} \sim N^{1/3}$$

and

$$\begin{aligned} t_R &\approx \frac{(Gmn)^{3/2}R^3}{8\pi[(\ln N)/3](Gmn)^{1/2}(Gm)^{3/2}n^{1/2}} \\ &\approx \frac{nR^3}{10\ln N}t_{\rm cr} \sim \frac{N}{10\ln N}t_{\rm cr} \end{aligned}$$

Application of this shows that in Globular star clusters the two-body relaxation time is $\sim 10^9$ y, so they are collisional systems. A galaxy like ours has the 2-body relaxation time much larger than the age of the Universe, and is hence collisionless.

Because a globular cluster system is collisional, tidal encounters between stars in it become important. The consequences of this manifest in binary formation and destruction, mass segregation (heavier objects sink to the centre of the cluster), extraction of energy from binary systems and feeding into the kinetic energy of cluster members etc. Binaries in globular clusters can be classified into "hard" and "soft". The former class has gravitational binding energy per star larger than the average kinetic energy of a cluster member, and the latter class has gravitational binding energy less than the kinetic energy of a cluster member. In stellar encounters, hard binaries become harder (more tightly bound) and soft binaries get softer (separation increases). Exchange of binary companions can also take place. A binary receives a recoil in an encounter, and may eventually be expelled from the stellar system due to this. Individual stars which have experienced gravitational encounters with binaries may also escape from the system if its velocity becomes larger than the local escape speed. Tidal effect due to the disk of the galaxy also is responsible for limiting the size of a globular cluster.