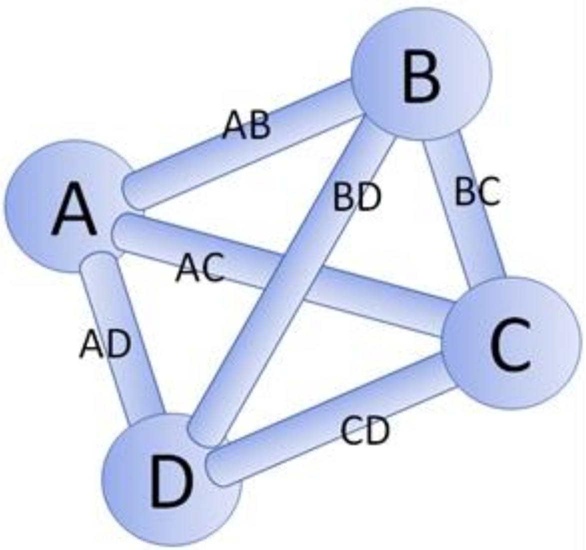
Assignment



Consider a fully connected network of 4 nodes shown on the right, the nodes being labelled A,B,C,D and the links designated AB,BC,CD, etc. (i.e., indicating the pair of nodes that each link connects).

(a) If each link can be either +ve or –ve, what is the total number of possible configurations that can be obtained (where the configurations can be represented as {+,+,+,+,+,+}, {+,+,+,+,+,–},{+,+,+,+,–,+}, {+,+,+,–,+,+}, etc.)?

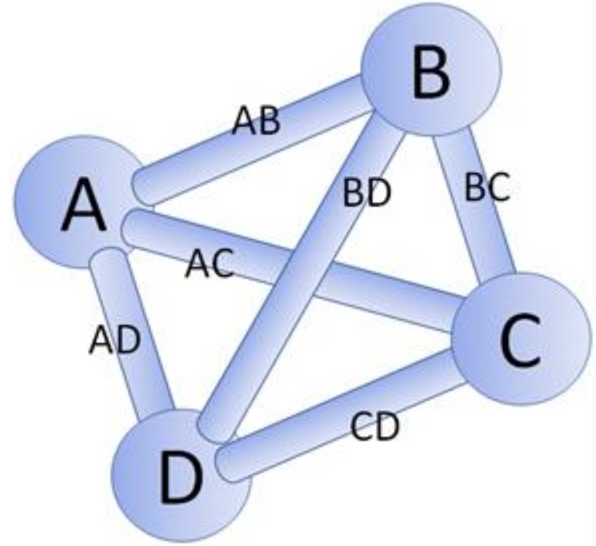
[Note that each of the links are distinct, i.e., AB being negative in a network where all other links are positive is a distinct configuration compared to one in which AD (for instance) is the only link which is negative.]

(b) How many distinct configurations will have 3 links positive and 3 links negative?

(c) Find how many of the total number of configurations with distinct assignment of link signs that you calculated in (a) are balanced. Note that a network is balanced if every closed loop or cycle is balanced, i.e., product of the signs of the links in the loop is + ve. However, instead of having to look at all 4-cycles as well as 3-cycles (triads), you can use the Cartwright-Harary theorem, according to which a fully connected network is balanced if each of the triads (ABC, ABD, etc.) are each individually balanced.

[Hint: find how many distinct triads are there in the network. If for a given configuration, even one of these triads is <u>not</u> balanced (i.e., has an odd number of negative links) the configuration will be <u>not</u> balanced.]

Answers



(a) The total number of possible configurations is $2^6 = 64$.

(b) Number of distinct configurations with 3 links positive and 3 links negative is ${}^6C_3 = 20$.

(c) Total number of configurations out of 64 which are balanced is 8.

Note: There are 4 triads to be considered ABC, ABD, BCD, ACD. You must ensure that <u>none</u> of these 4 triads have an odd number of negative signs, in order to be able to call the configuration balanced. The only ones which satisfy this criterion are:

I. If all links are positive, that single configuration is balanced.

II. If 3 links are positive, 3 links are negative, then out of 20 possible configurations, 4 are balanced: 1. Links AB,AC,AD are negative, rest positive; 2. Links AB,BC,BD are negative, rest positive; 3. Links AC,BC,CD are negative, rest positive; 4. Links AD,BD,CD are negative, rest positive.

III. If 2 links are positive, 4 links are negative, then, out of the 15 possible configurations, 3 are balanced: 1. Config with links AB and CD positive, the rest negative; 2. Config with AC and BD positive, the rest negative, 3. Config. with AD and BC positive, the rest negative