Collective behaviour of a family of power law models

Manas Kulkarni

(ICTS-TIFR, Bangalore)

INTERNATIONAL CENTRE *for* THEORETICAL SCIENCES

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

Collaboration

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Saikat Santra (ICTS)

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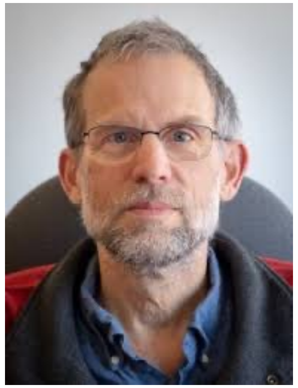
Anupam Kundu (ICTS)

Satya Majumdar (Paris-Saclay)

Gregory Schehr (Sorbonne, Paris)

David Mukamel (Weizmann)

Bhanu Kiran (ICTS)



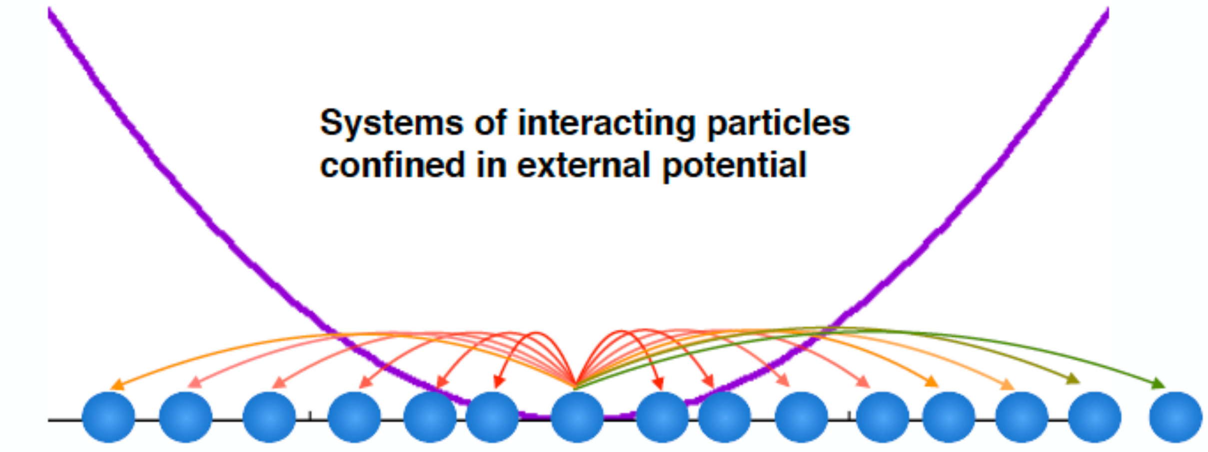
David Huse (Princeton)

- S. Agarwal, A. Dhar, **MK**, A. Kundu, S. N. Majumdar, D. Mukamel, G. Schehr, PRL (2019)
- S. Agarwal, **MK**, A. Dhar, J. Stat Phys (2019)
- A. Kumar, **MK**, A. Kundu, PRE (2020)
- J. Kethepalli, **MK**, A. Kundu, S. N. Majumdar, D. Mukamel, G. Schehr, J. Stat Mech (2021, 2022)
- S. Santra, J. Kethepalli, S. Agarwal, A. Dhar, **MK**, A. Kundu, PRL (2022)
- B. Kiran, D. A. Huse, **MK**, PRE (2021)

+ some ongoing works
(equilibrium and dynamics)

Contents

Definitions and Questions



Courtesy of Anupam Kundu (ICTS)

- Common in physical systems
- All-to-all pairwise repulsive interaction

Ensures repulsion

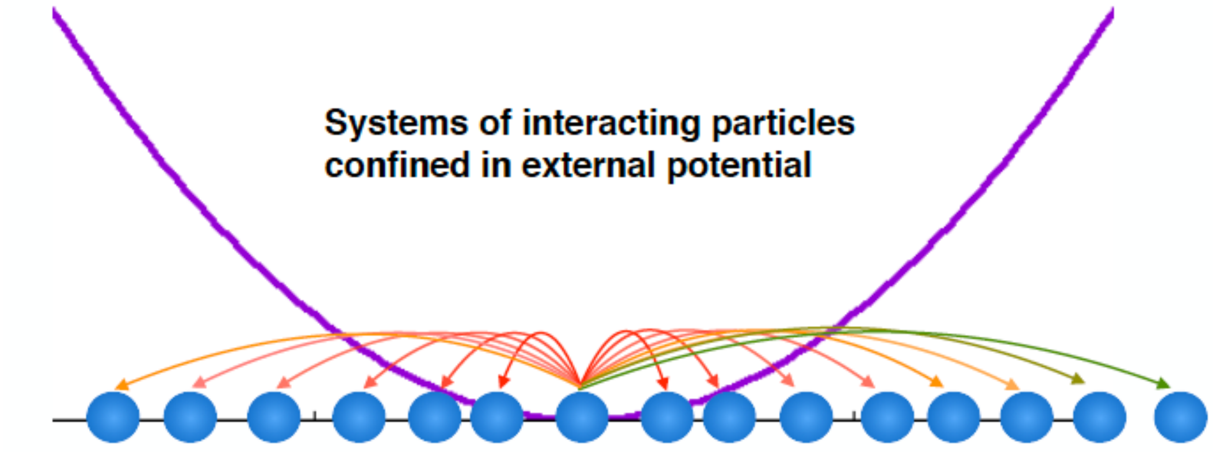
$$E(\{x_i\}) = \frac{1}{2}\sum_{i=1}^{N} V_{ex}(x_i) + \frac{J\,\mathrm{sgn}(k)}{2}\sum_{j\neq i} V_{int}(|x_i - x_j|); \quad J > 0$$

$$\text{s.t.} \quad V_{ex}(x)|_{|x|\to\infty} \to \infty, \quad V_{int}(r)|_{r\to 0} \sim \frac{1}{r^k}$$

Power-law exponent

Definitions and Questions

- Common in physical systems
- All-to-all pairwise repulsive interaction



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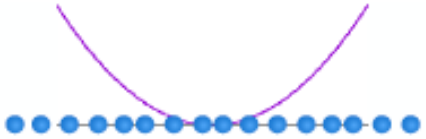
Power-law exponent

For external trapping potential

$V_{ex}(x) = x^2$	Harmonic	(very common)
$V_{ex}(x) = x^4$	Quartic	(Shin, MIT, Thesis, 2006)
$V_{ex}(x) = x^4 - x^2$	Double well	
$V_{ex}(x) = \cosh(x)$	Box-like	(Hadzibabic Lab, Cambridge)

Definitions, Statement and Questions

Zero/Very lowTemperature



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Interplay between two terms:

Confining trap and repulsive interaction

Interesting questions:

- Configuration of particles x_i ‘s that minimizes energy ?
- Macroscopic density in large-N limit ?
- Large-N field theory ?

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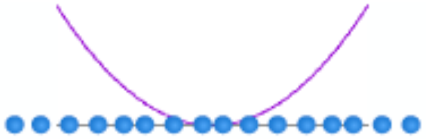

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Finite/Considerable Temperature

Equilibrium joint probability density function at temperature T

$$P(x_1, \cdots, x_N) = \frac{1}{Z_N(\beta)} e^{-\beta E[\{x_i\}]}$$

$$Z_N(\beta) = \int \prod_{i=1}^{N} dx_i \, e^{-\beta E[\{x_i\}]}$$

Competition between confining trap, repulsive interaction and entropy

Empirical density

$$\hat{\rho}_N(x) = \frac{1}{N}\sum_{i=1}^{N} \delta(x - x_i)$$

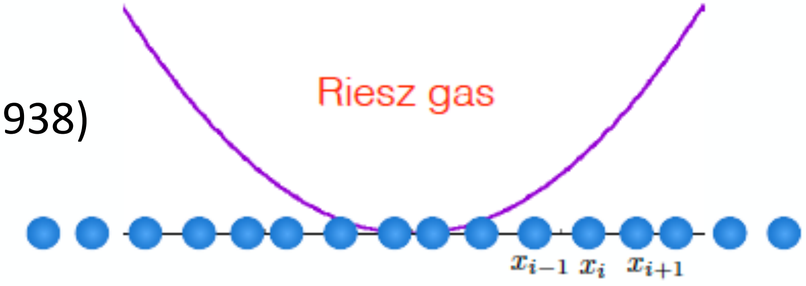
What is $\langle \hat{\rho}_N(x) \rangle$?

Thermal average of the empirical density

Motivation

Many interesting special values of k

M. Riesz (1938)



Satya’s Talk

Let us take a specific family of models:

Harmonic potential and power law interaction

$$V_{ex}(x) = x^2 \qquad V_{int}(r) = |r|^{-k}$$

$$E(\{x_i\}) = \frac{1}{2}\sum_{i=1}^{N} x_i^2 + \frac{J\ \mathrm{sgn}(k)}{2}\sum_{j\neq i}\frac{1}{|x_i - x_j|^k}; \quad J > 0$$

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Specific Examples

First Example **(1dOCP, k= -1)**

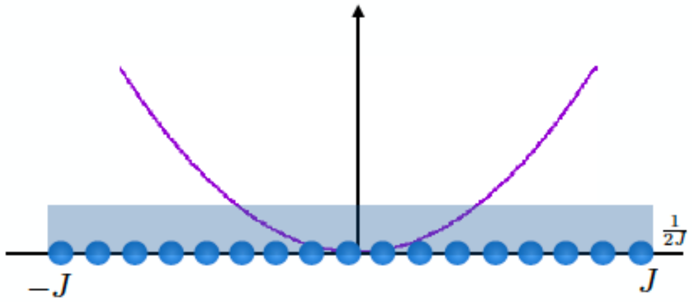
k=-1: one dimensional one component plasma **(1dOCP)**

$$E(\{x_i\}) = \frac{1}{2}\sum_{i=1}^{N} x_i^2 - \frac{J}{2}\sum_{j\neq i}|x_i - x_j|; \quad J > 0$$

Negatively charged particles with pairwise Coulomb interaction (linear in 1D) in the uniform background of positive charges — ensuring neutrality

$$L_N \sim O(N)$$

$$\langle \rho_N(x)\rangle = \frac{1}{N}\rho_f\left(\frac{x}{N}\right), \quad \rho_f(y) = \frac{1}{2J}$$



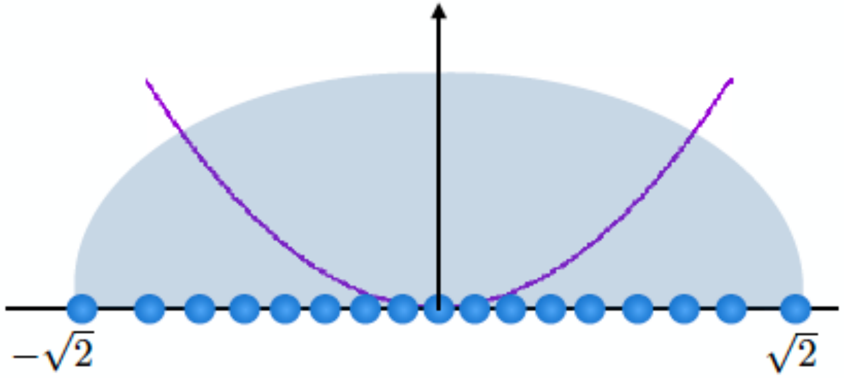
Second Example **Dyson's Log Gas** $k \to 0$

$$E(\{x_i\}) = \frac{1}{2}\sum_{i=1}^{N} x_i^2 + \frac{J\,\mathrm{sgn}(k)}{2}\sum_{j\neq i}\frac{1}{|x_i - x_j|^k}; \quad J > 0$$

For k ⟶ 0 $|x_i - x_j|^{-k} \approx 1 - k\ln|x_i - x_j|$ and set $J = \frac{1}{k}$

$$E(\{x_i\}) = \frac{1}{2}\sum_{i=1}^{N} x_i^2 - \sum_{j\neq i}\ln|x_i - x_j|$$

Dyson's Log-gas



Wigner Semi-circle

$$L_N \sim O(\sqrt{N})$$

- Minima located at zeros of N'th Hermite polynomial
- Non-interacting trapped Fermions
- Eigenvalue distribution in Random Matrix Theory
- Relation to KPZ universality class
- Algebraic Stieltjes problem

ICTS youtube lectures by Satya: *RMT and its Applications (BSSP 2019)*
S. N. Majumdar and G. Schehr (J. Stat Mech 2014)

Gregory's talk

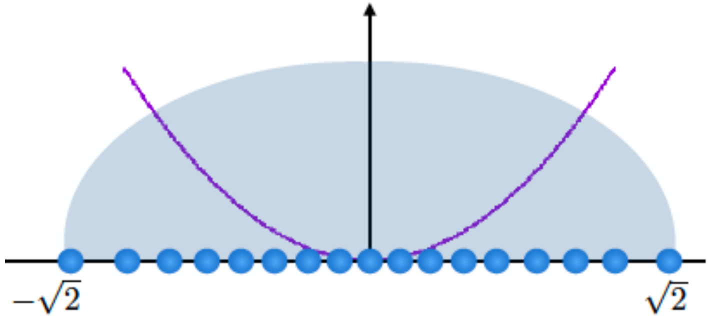
Third Example **(Calogero-Moser System, k=2)**

Herbert's talk

$$E(\{x_i\}) = \frac{1}{2}\sum_{i=1}^{N} x_i^2 + J\sum_{j\neq i}\frac{1}{|x_i - x_j|^2}; \quad J > 0$$

$$\langle \hat{\rho}_N(x)\rangle = \frac{1}{\sqrt{N}}\rho_{sc}\left(\frac{x}{\sqrt{N}}\right), \quad \rho_{sc}(y) = \frac{1}{\pi}\sqrt{2-y^2}$$

Wigner semi-circle too !



- Integrable even in external confining potentials (upto quartic polynomial potentials)
- Itself appears in various branches of physics

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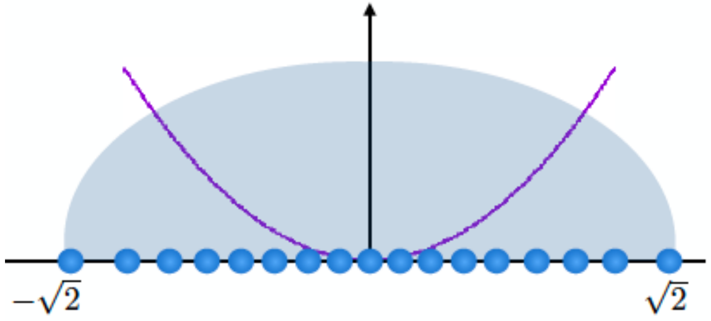


- Integrable even in external confining potentials (upto quartic polynomial potentials)
- Itself appears in various branches of physics

$$L_N \sim O(\sqrt{N})$$

Many other examples

$$E(\{x_i\}) = \frac{1}{2}\sum_{i=1}^{N} x_i^2 + \frac{J\,\mathrm{sgn}(k)}{2}\sum_{j\neq i}\frac{1}{|x_i-x_j|^k}$$

k=1 (3D Coloumb confined in 1D)

k=3 (Dipolar gas)

k=4 (Charge induced quadrapole interaction)

k=5 (quadrapole - quadrapole interaction)

$k \to \infty$ (Hard Rods), Anupam’s Talk

- Harmonic trap is most ubiquitous in experiments
- Absorption images are very well developed to observe collective dynamics
- Cutting edge techniques that resolve at the level of the single particle

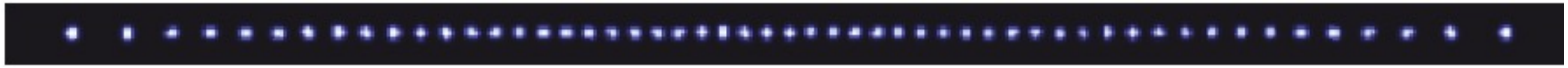
Rotational spectroscopy of diatomic molecules, Brown and Carrington
Quantum Gas Microscope, Bakr et al, Nature 2009
Dipolar collisions of polar molecules, Ni et al, Nature (2010)
Dipolar Interactions, Griesmaier, PRL (2008)
Cold Atoms, **M.K.** & Abanov, PRA (2012)

Cold trapped ions

$$E(\{x_i\}) = \frac{1}{2}\sum_{i=1}^{N} x_i^2 + \frac{J\,\mathrm{sgn}(k)}{2}\sum_{j\neq i}\frac{1}{|x_i - x_j|^k}$$

Zhang et al, Nature, 2017

$^{171}\mathrm{Yb}^{+}$ ion



- Chain of 53 ions
- Experiment - Bright spots indicate the location of ions
- Approximately think of this as k ~ 1 case with external Harmonic trap
- Efforts are on the scale-up the system
- For ion experiments - In principle 0<k<3, but in practice, currently restricted to 0.5 <k < 1.8

Programmable quantum simulations of spin systems with trapped ions

C. Monroe et al, RMP (2021)

Promising experimental avenues to realize low dimensional long ranged interacting systems

(Experiments on long ranged interactions in RRI)

Cold trapped ions

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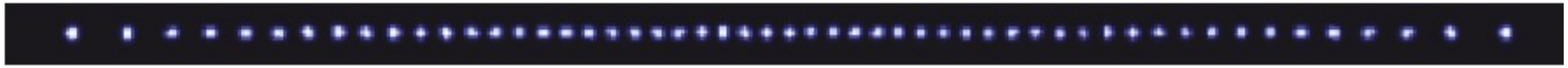

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Promising experimental avenues to realize low dimensional long ranged interacting systems

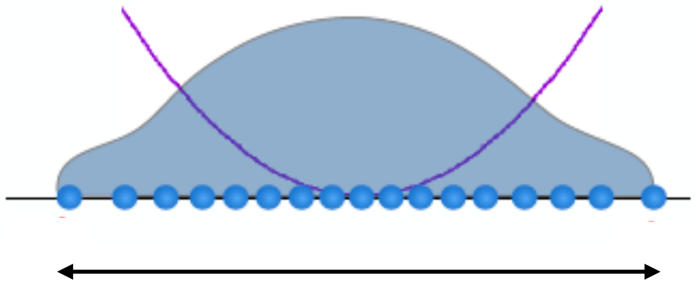
(Experiments on long ranged interactions in RRI)

Recall the questions for general k:

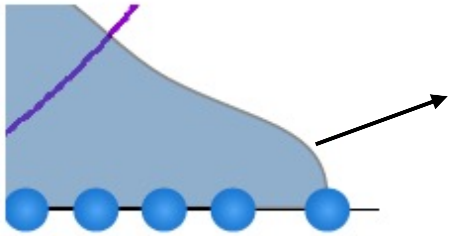
- How does shape of density profile change when one tunes k ?
- What is the large-N field theory for general k ?

Answers

S. Agarwal, A. Dhar, **M.K**, A. Kundu, S. N. Majumdar
D. Mukamel, G. Schehr, PRL (2019)



Size of the cloud $L_N \sim O\left(N^{\alpha_k}\right)$



How density behaves at the edges ?
γ_k

$$\langle \rho_N(x)\rangle \approx \frac{1}{\ell_k\, N^{\alpha_k}}\, F_k\left(\frac{x}{\ell_k\, N^{\alpha_k}}\right)$$ where $$F_k(z) = \frac{1}{B(\gamma_k+1, \gamma_k+1)} \left(\frac{1}{4} - z^2\right)^{\gamma_k}$$

-2 $\leq k \leq 1$	$k > 1$
$\alpha_k = \frac{1}{k+2}$	$\alpha_k = \frac{k}{k+2}$
$\gamma_k = \frac{k+1}{2}$	$\gamma_k = \frac{1}{k}$

Answers

S. Agarwal, A. Dhar, **M.K**, A. Kundu, S. N. Majumdar
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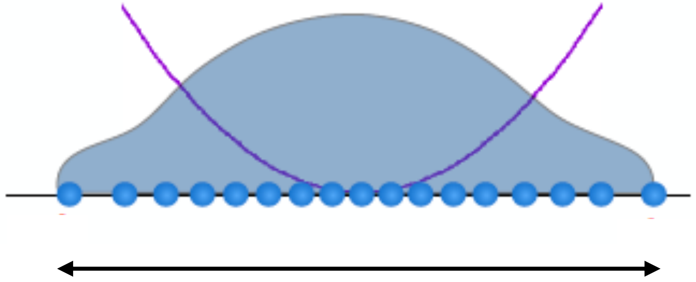

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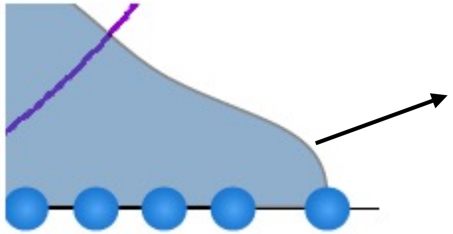

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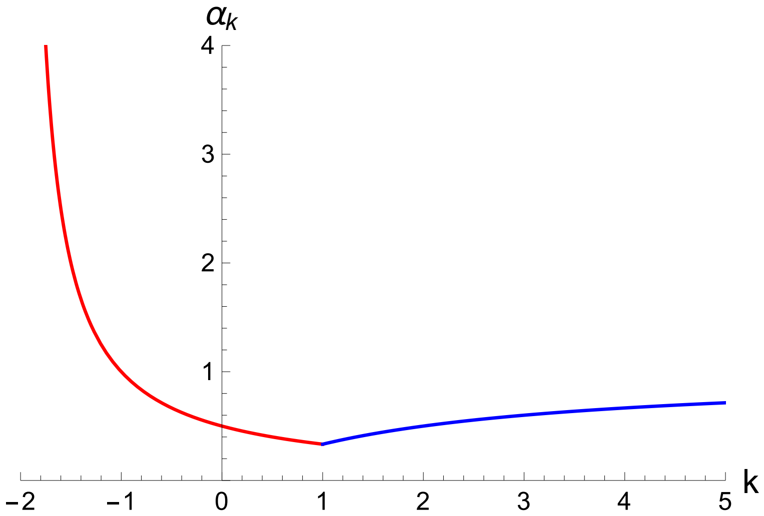
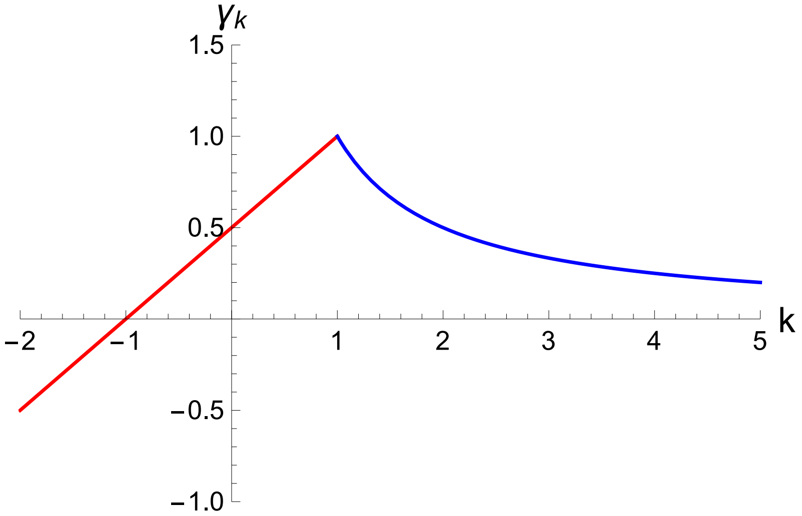
Interesting non-monotonic behavior in $\alpha_k,\ \gamma_k$

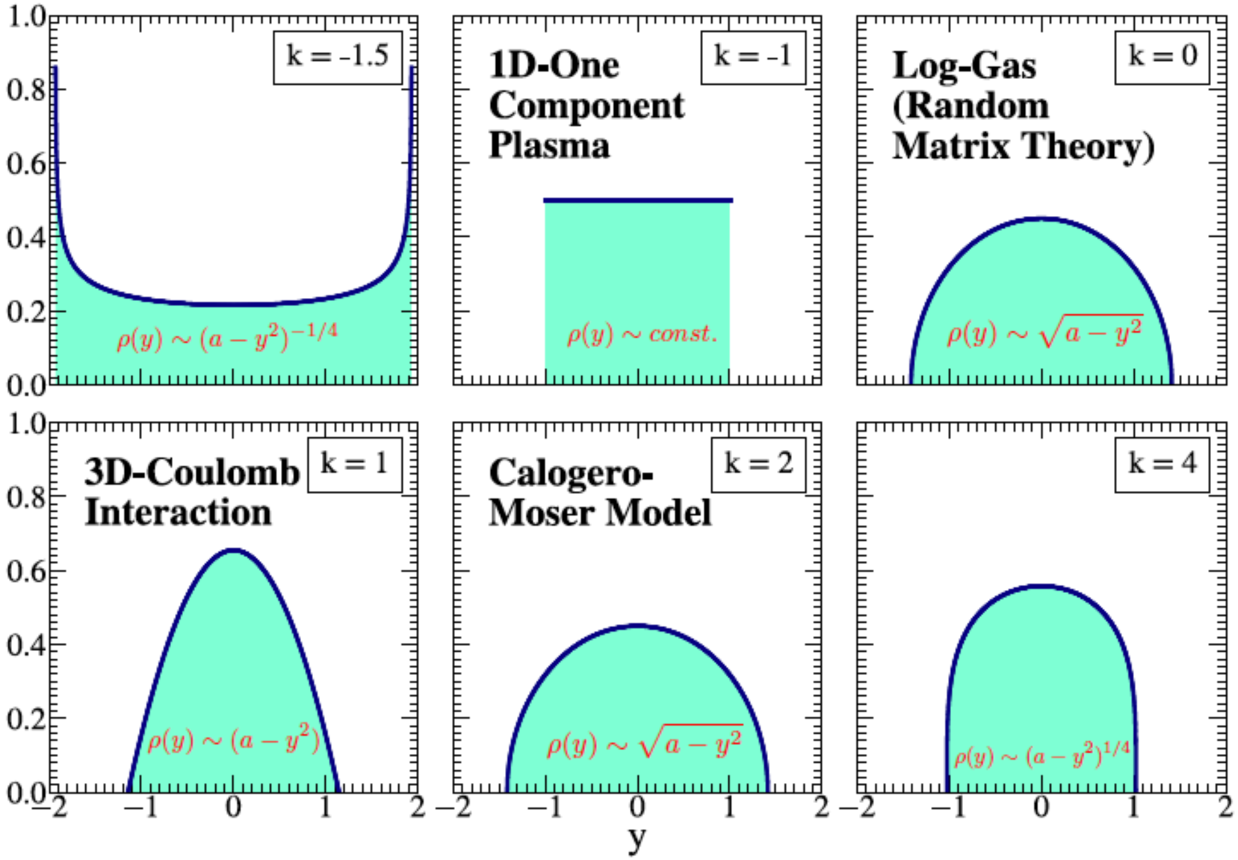


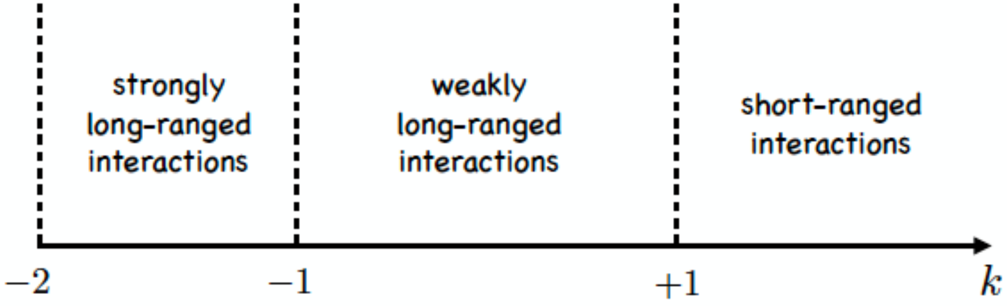


Numerical Verification:

Answers

Sanaa Agarwal





strongly long-ranged interactions

weakly long-ranged interactions

short-ranged interactions

These densities are obtained by minimising some action through a saddle point calculation for large N

$$S_{N,\mu}[\rho(z)] = \beta\ \underbrace{\mathcal{E}_N[\rho(z)]}_{energy} - \underbrace{N\int dz\rho(z)\ln\rho(z)}_{entropy} + \underbrace{\mu\left(\int \rho(z)dz - 1\right)}_{normalisation}$$

$$E(\{x_i\}) = \frac{1}{2}\sum_{i=1}^{N} x_i^2 + \frac{J\ \mathrm{sgn}(k)}{2}\sum_{j\neq i}\frac{1}{|x_i-x_j|^k} \quad \Rightarrow \quad \mathcal{E}_N[\rho(z)]$$

Answers

Note: Every particle was paired with every other particle [all-to-all coupling]

S. Agarwal, A. Dhar, **M.K**, A. Kundu, S. N. Majumdar, D. Mukamel, G. Schehr, PRL (2019)

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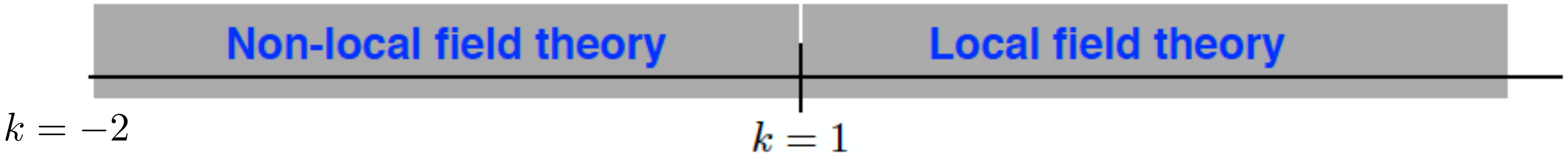
$$\mathcal{E}_N[\rho(x)] = \frac{N}{2}\int dx\ V_{ex}(x)\rho(x)$$
$$+\begin{cases} J\ \mathrm{sgn}(k)\ N^2 \int dx \int dy\ \rho(x)V_{int}(|x-y|)\rho(y), & for\ k<1 \\ J\ N^2 \ln N \int dx\ \rho(x)^2, & for\ k=1 \\ J\ \zeta(k)\ \ N^{1+k}\int dx\ \rho(x)^{k+1}, & for\ k>1 \end{cases}$$

Riemann Zeta Function

$$\zeta(k) = \sum_{n=1}^{\infty}\frac{1}{n^k}$$



Non-local field theory

Local field theory

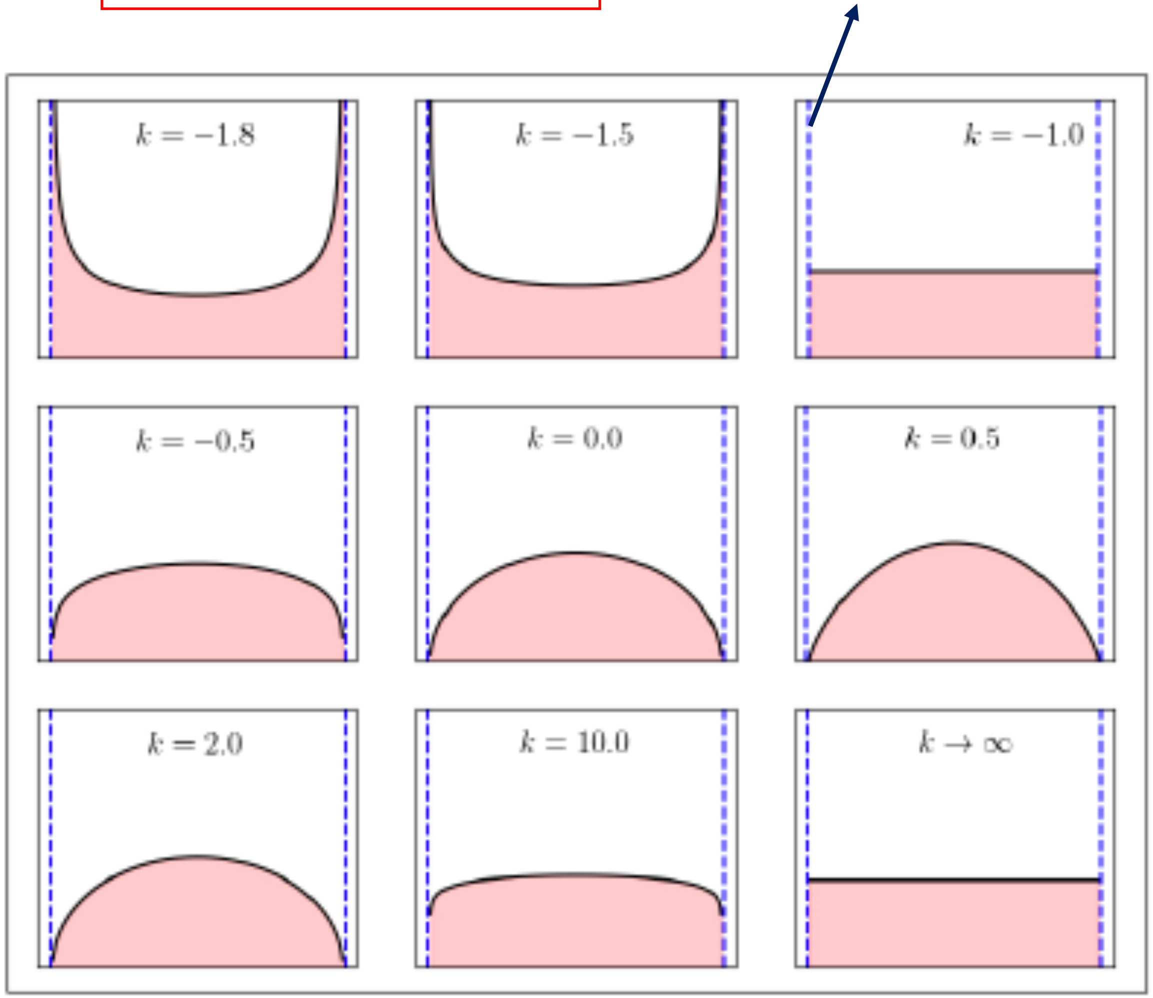
$k=-2$

$k=1$

What happen if we put a barrier/wall ?

Recap: Without Wall

Vertical blue lines just indicate the finite support



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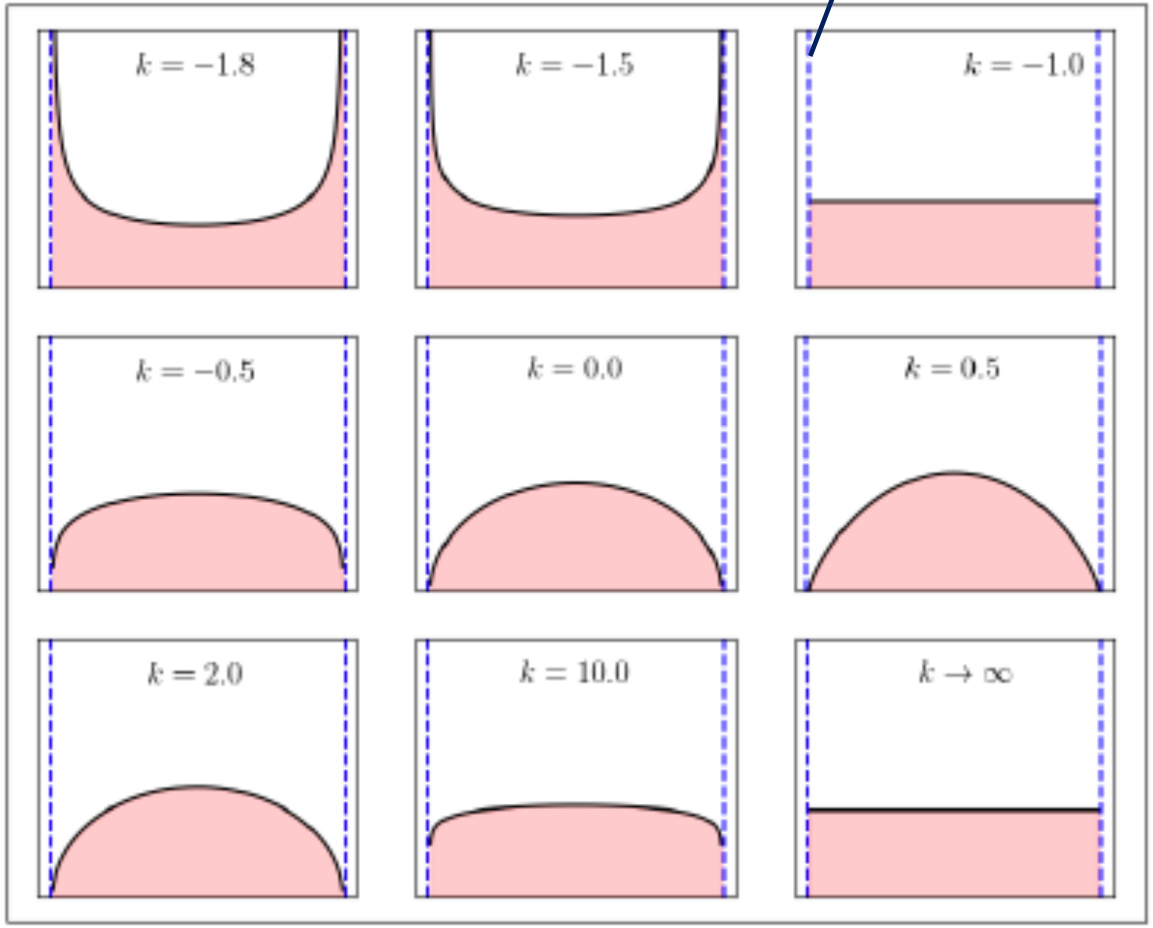
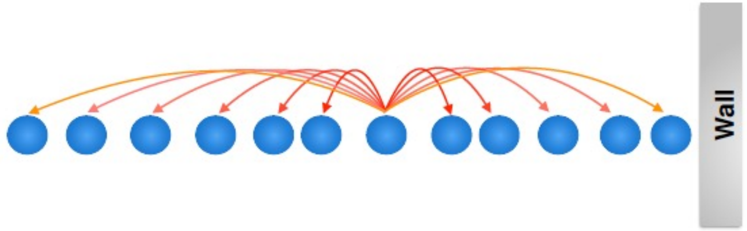


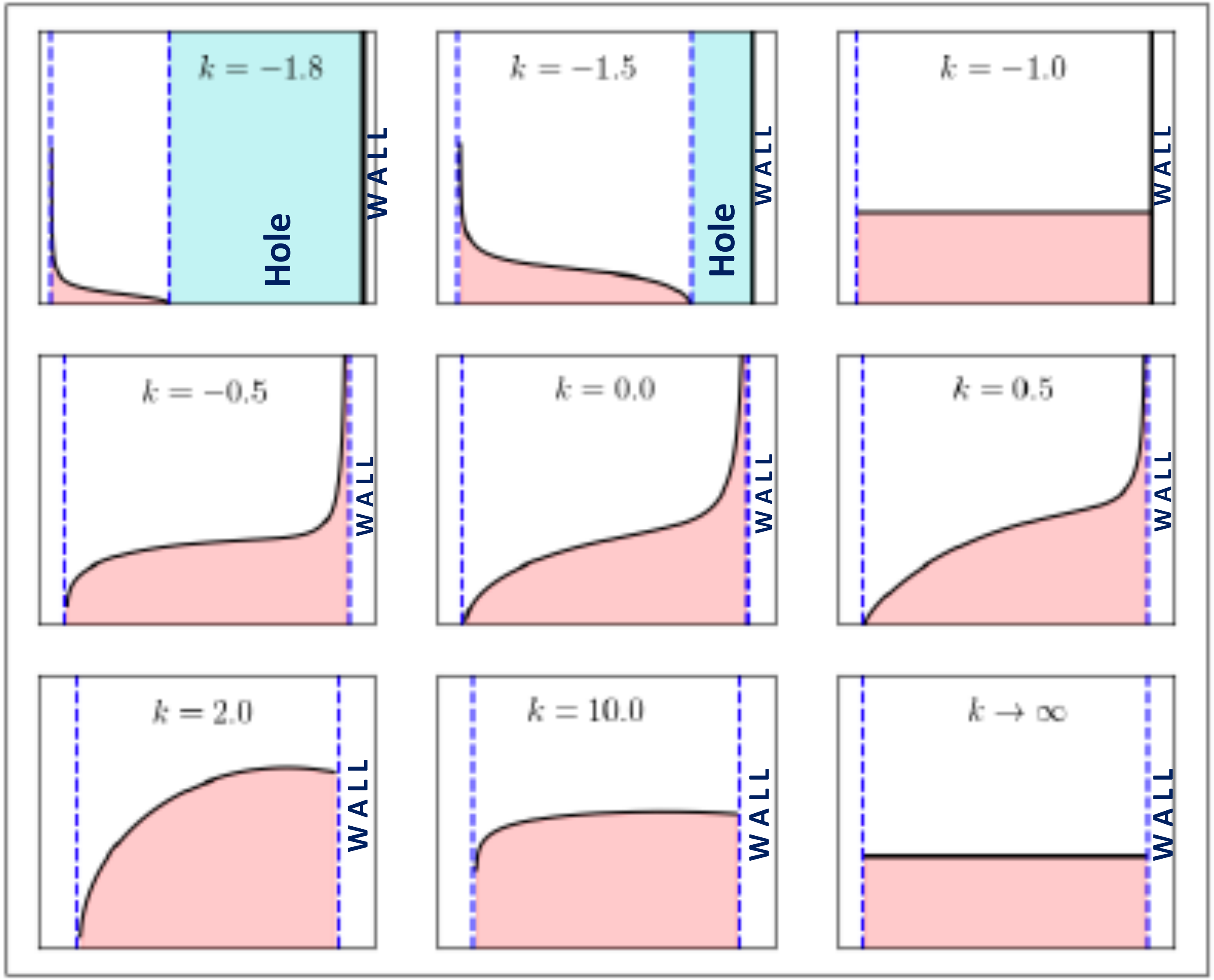


Courtesy of Anupam Kundu (ICTS)

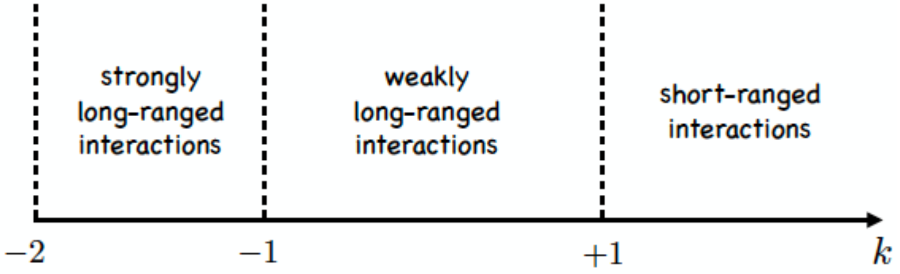
Why is barrier interesting ?

- Experimentally feasible
- Computation of extreme value statistics

Density profiles in the presence of a barrier



Jitendra Kethepalli (ICTS)



<u>k>1</u> - Truncated dome
- Finite support on the left

<u>-1<k<1</u> - Integrable divergence at wall
- Finite support on the left

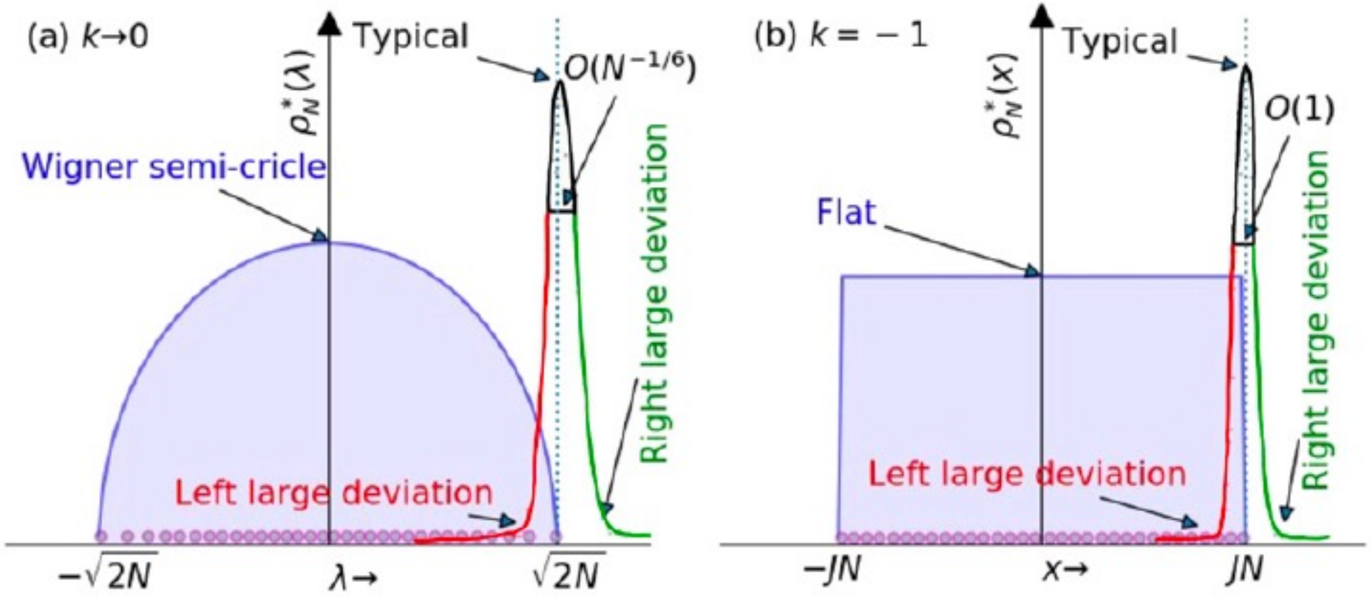
<u>-2<k<-1</u> - Exotic density profile
- Two disjoint pieces separated by a hole
- Integrable divergence on the left
- Finite support on the right of extended piece

Extreme value statistics (EVS)

J. Kethepalli, **M.K.** , A. Kundu, S. N. Majumdar, D. Mukamel, G. Schehr, J. Stat Mech (2022)

- Distribution of position of the right most particle x_{max} in a N particle Riesz gas

Let us recall some special cases of Riesz gas



Several EVS works by Dean, Majumdar, Schehr, Evans, Comtet, Forrester etc.

Correlated Random variables: S. N. Majumdar, A. Pal, G. Schehr, Physics Reports (2020)

Dyson's Log Gas: D. Dean, S. N. Majumdar (PRL 2006, PRE 2008), S. N. Majumdar and G. Schehr (J. Stat Mech 2014)

1d OCP: A. Dhar, A. Kundu, S. N. Majumdar, S. Sabhapandit, G. Schehr (PRL, 2017, JPhysA 2018)

Correlated simultaneous resetting/quenched gas: Biroli, Larralde, Majumdar, Schehr (PRL 2023, PRE 2023)
Biroli, **M. K.**, Majumdar, Schehr (arXiv 2023)

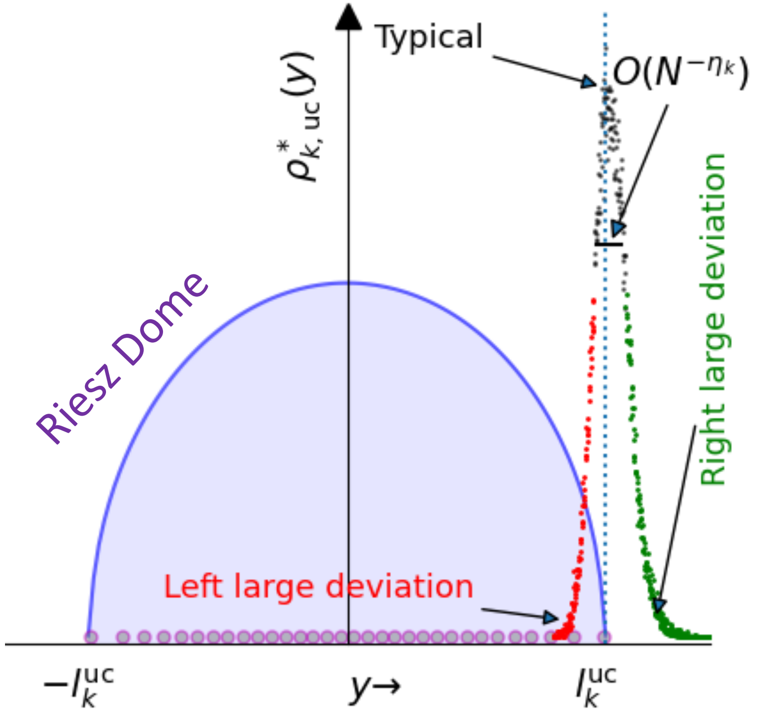
Jitendra Kethepalli (ICTS)

Extreme value statistics: Results of the Riesz gas

Left large deviation function

$$\text{Prob.}\,[y_{\max} < w, N] \approx \begin{cases} e^{-\beta N^{2\alpha_k+1}\Phi_-(w,k)} & l_k^{\text{uc}} - w \gtrsim O(1) \\ \mathcal{F}_\beta^{(k)}\left(N^{\eta_k}\left(w - l_k^{\text{uc}}\right)\right) & |w - l_k^{\text{uc}}| \lesssim O(N^{-\eta_k}) \\ 1 - e^{-\beta N^{2\alpha_k}\Phi_+(w,k)} & w - l_k^{\text{uc}} \gtrsim O(1), \end{cases}$$

Right large deviation function

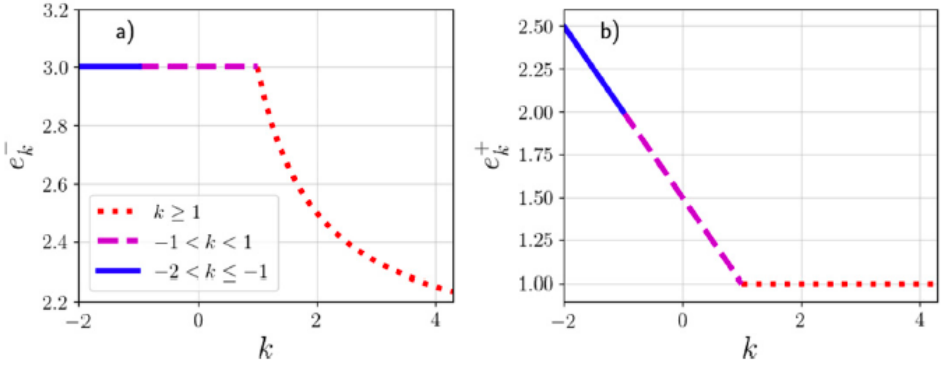


Asymptotics of large deviation function

$$\Phi_-(w \to l_k^{\text{uc}-}, k) \propto (l_k^{\text{uc}} - w)^{e_k^-}$$

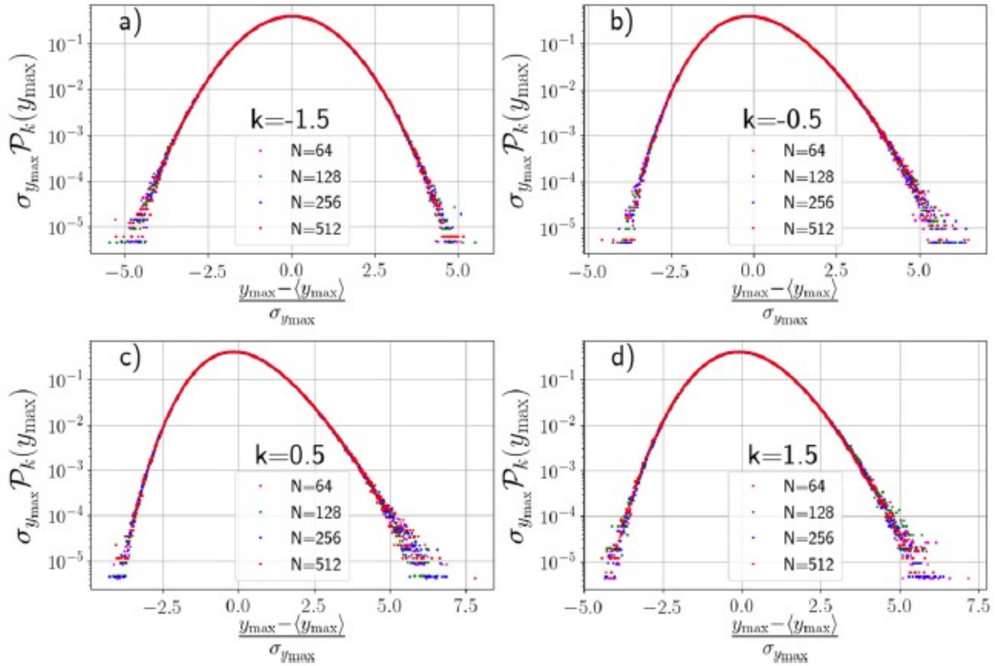
$$\Phi_+(w \to l_k^{\text{uc}+}, k) \propto (w - l_k^{\text{uc}})^{e_k^+}$$

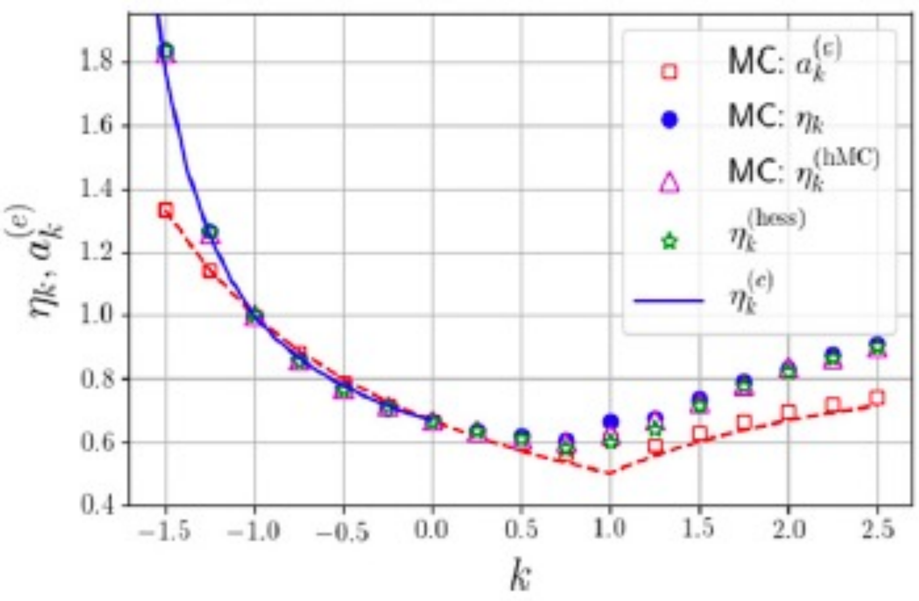
Range of interaction	e_k^-	e_k^+
Short-range: k>1	2+1/k	1
Weakly long-range: -1<k<1	3	(3-k)/2
Strongly long-range: -2<k<-1	3	(3-k)/2



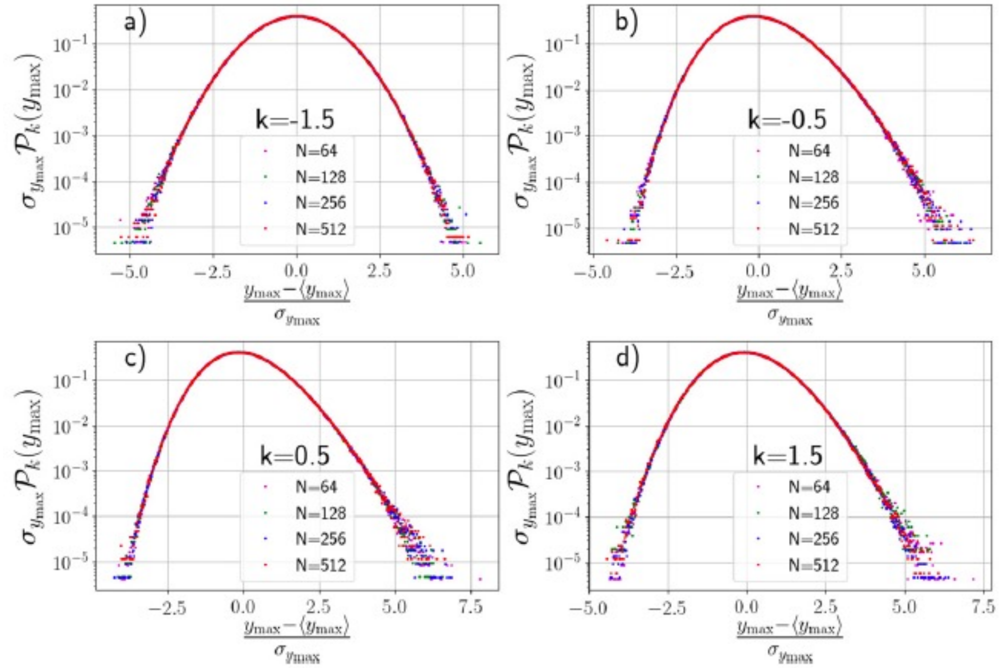
Third order phase transition in the Riesz gas !

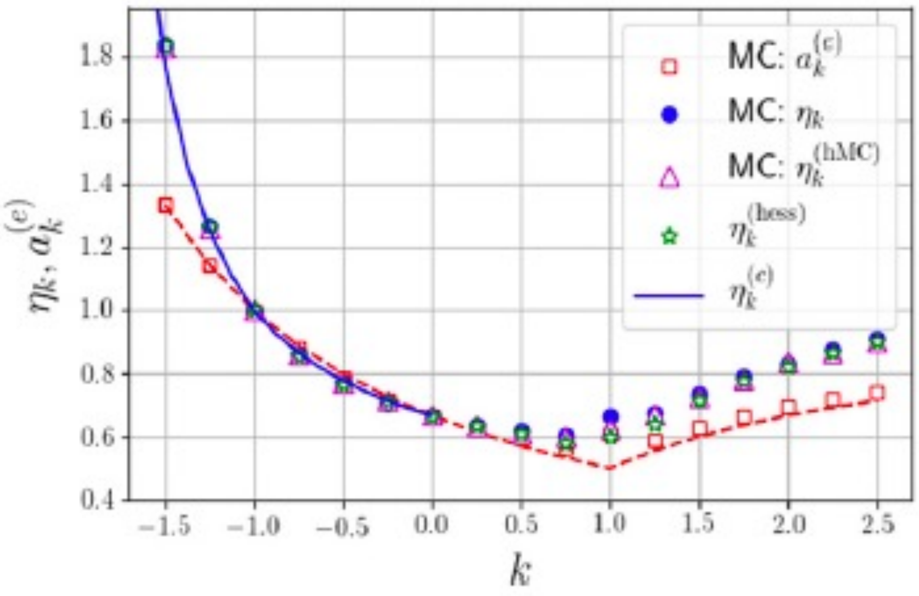
Typical fluctuations for rightmost particle: Riesz gas





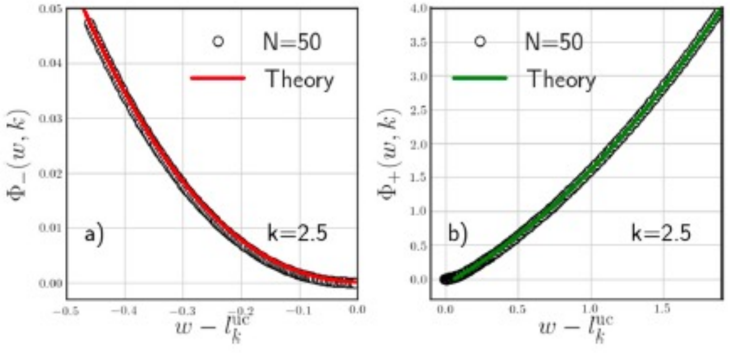
Typical fluctuations for rightmost particle: Riesz gas



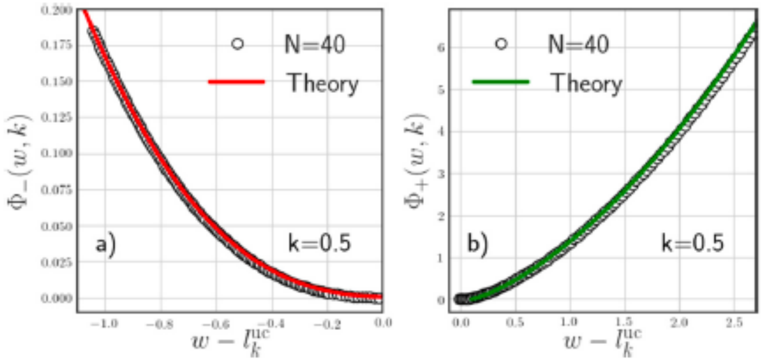


Large Deviation Function for rightmost particle: Riesz gas

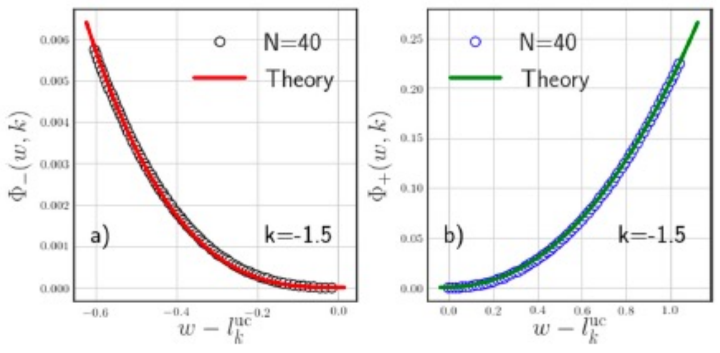
Short-range k > 1



Weakly long range -1 < k < 1



Strongly long range -2 < k < -1

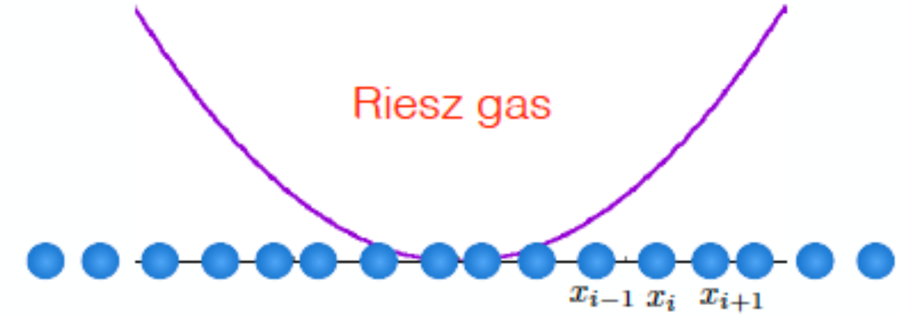


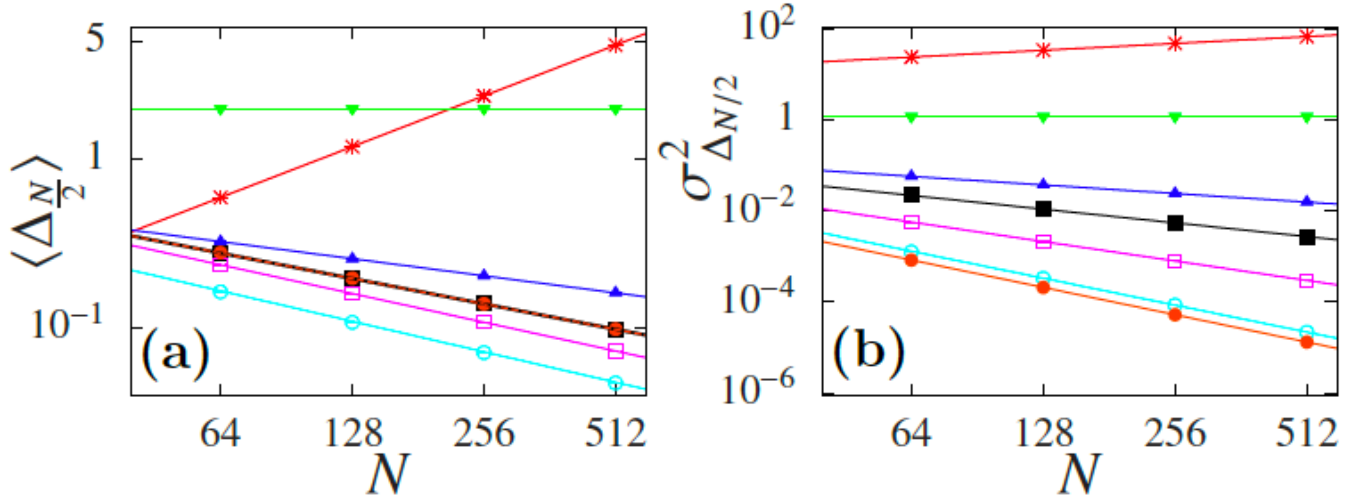
Saikat Santra (ICTS)

Jitendra Kethepalli (ICTS)

What about bulk gap ?

S. Santra, J. Kethepalli, S. Agarwal, A. Dhar, **MK**, A. Kundu, PRL (2022)





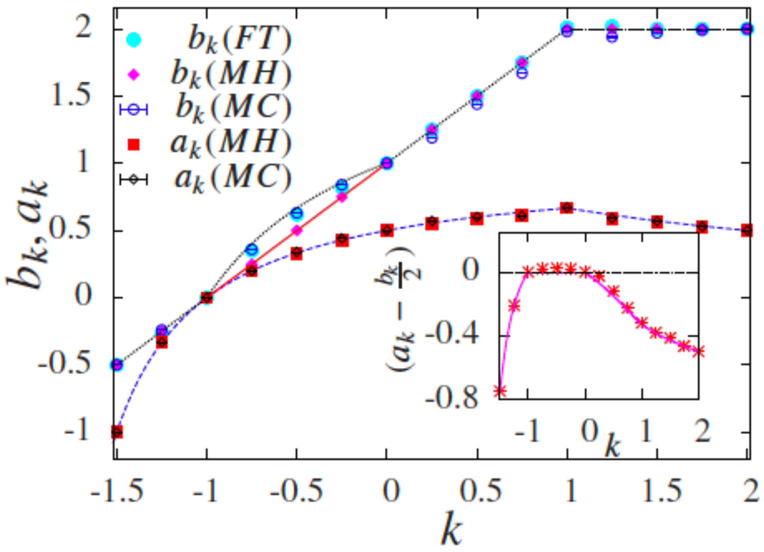
Let us stick to bulk: $1 \ll i \ll N-1$

Interparticle Separation: $\Delta_i = x_{i+1} - x_i$

$$\boxed{\langle \Delta_i \rangle \sim N^{-a_k}}$$

gap fluctuations $\sigma^2_{\Delta_i} = \langle \Delta_i^2 \rangle - \langle \Delta_i \rangle^2$

$$\boxed{\sigma^2_{\Delta_{N/2}} \sim N^{-b_k}}$$



$$a_k = \begin{cases} \frac{k+1}{k+2} & \text{for } -2 < k < 1 \\ \frac{2}{k+2} & \text{for } k > 1 \end{cases}$$

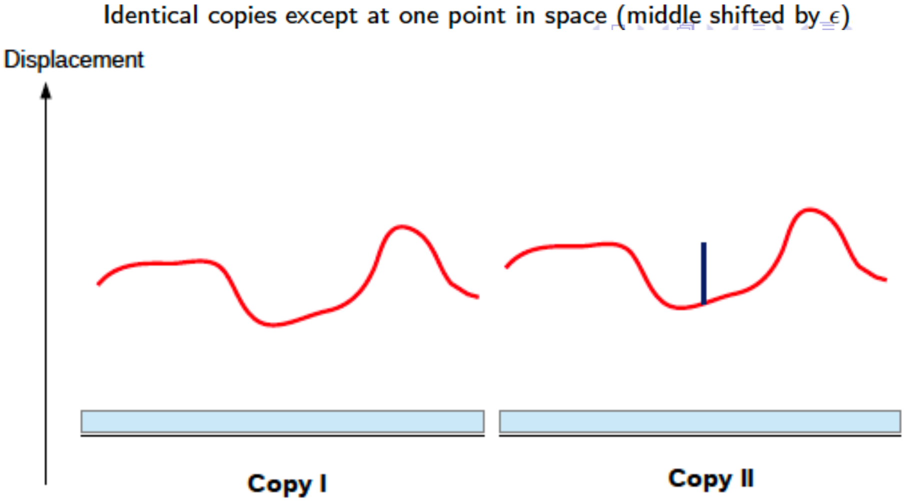
Below is a conjecture:

$$b_k = \begin{cases} 1+k & \text{for } \quad -2 < k < -1 \\ 2(k+1)/(k+2) & \text{for } \quad -1 < k < 0 \\ 1+k & \text{for } \quad 0 < k < 1 \\ 2 & \text{for } \quad k > 1. \end{cases}$$

A. Flack, S. N. Majumdar, G. Schehr (Gap, FCS, etc in 1dOCP)

Spatio-temporal spread of perturbations at very low temperatures

B. Kiran, D. A. Huse, **MK** (PRE, 2021)



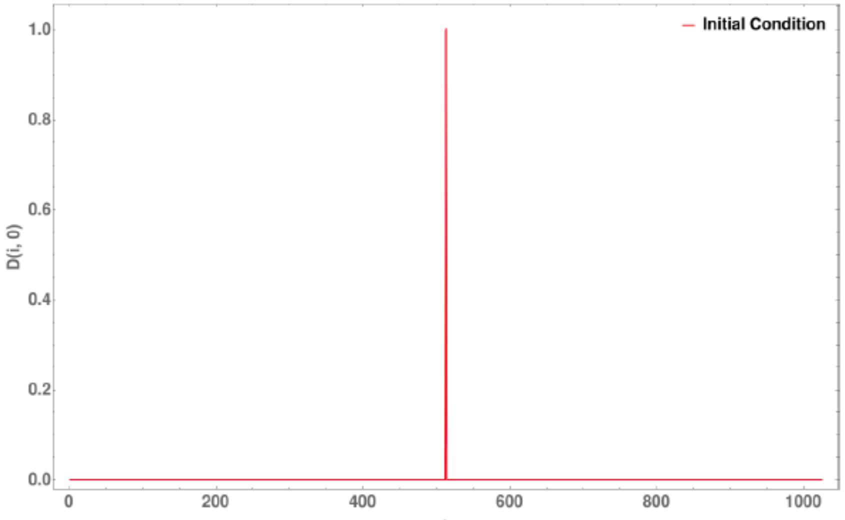
Das *et al*, PRL (2018)

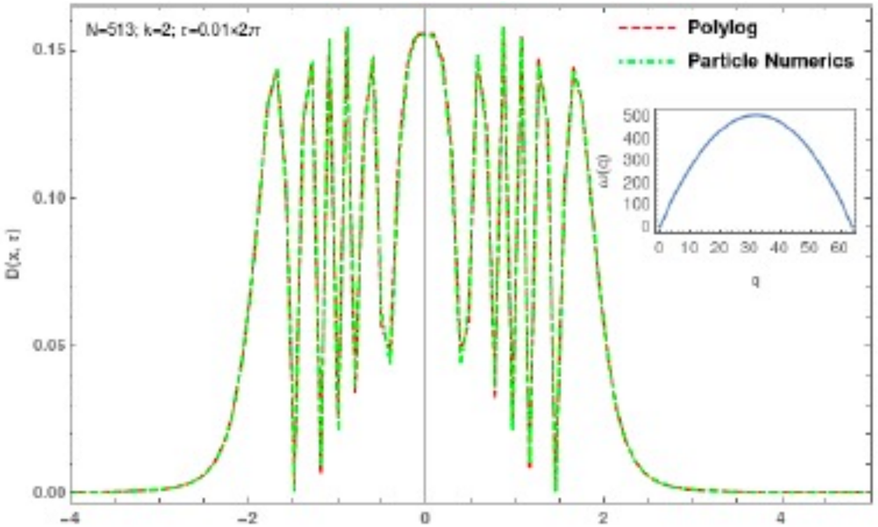
$$\dot{x}_i = p_i/m$$

$$\dot{p}_i = -m\omega^2 x_i + \frac{Jk}{2}\sum_{j\neq i}\frac{\mathrm{sgn}(x_i - x_j)}{|x_i - x_j|^{k+1}}$$

$$D(i,t) = \left|\frac{x_i^I(t) - x_i^{II}(t)}{x_0^I(0) - x_0^{II}(0)}\right|^2 \equiv \left|\frac{\delta x(t)}{\epsilon}\right|^2$$

Bhanu Kiran (ICTS-TIFR)





OTOC says how perturbations spread in space and grow / decay in time

General algorithm

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + V_k(\{x_j\})$$

$$V_k(\{x_j\}) = \sum_{i=1}^{N} \left[\frac{m\omega^2}{2} x_i^2 + \frac{J}{2} \sum_{j \neq i} \frac{1}{|x_i - x_j|^k} \right]$$

$$\dot{x}_i = p_i / m$$

$$\dot{p}_i = -m\omega^2 x_i + \frac{Jk}{2} \sum_{j \neq i} \frac{\mathrm{sgn}(x_i - x_j)}{|x_i - x_j|^{k+1}}$$

$$\omega_k(q) = \sqrt{\frac{Jk(k+1)}{ma^{k+2}} \left[2\zeta(k+2) - P(k,q) \right]}$$

$$\omega_k(q) \approx \begin{cases} \alpha_k q - \beta_k q^k, & 1 < k < 3 \\ \alpha_k q + \gamma_3 q^3 \log(qa), & k = 3 \\ \alpha_k q - \delta_k q^3 - \beta_k q^k, & 3 < k < 5 \end{cases}$$

$$D(x,t) = \left| \frac{a}{2\pi} \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dq \cos\Big(qx - \omega_k(q)t \Big) \right|^2$$

Plot and analyze OTOC and its asymptotics (either with exact dispersion or expansion)

General algorithm

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + V_k(\{x_j\})$$

$$V_k(\{x_j\}) = \sum_{i=1}^{N} \left[\frac{m\omega^2}{2} x_i^2 + \frac{J}{2} \sum_{j \neq i} \frac{1}{|x_i - x_j|^k} \right]$$

$$\dot{x}_i = p_i / m$$

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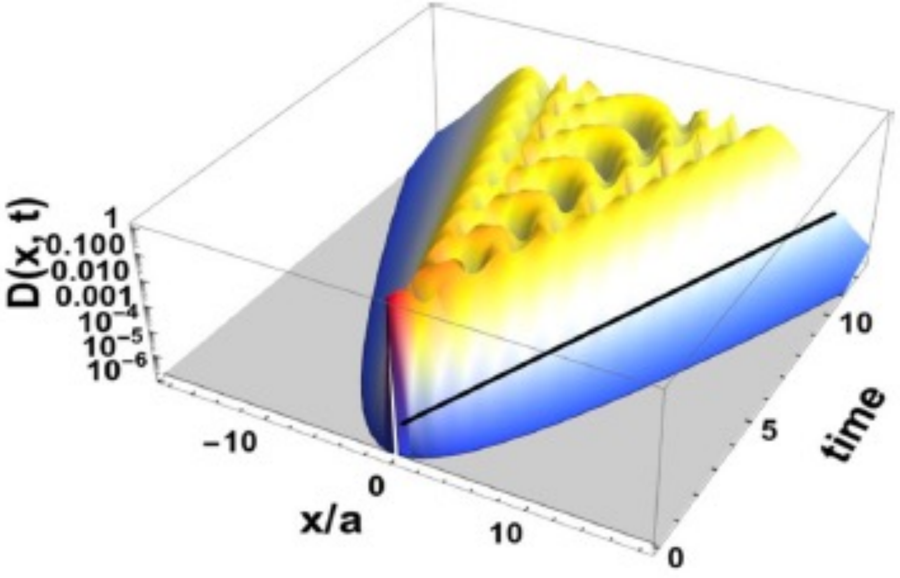
Plot and analyze OTOC and its asymptotics
(either with exact dispersion or expansion)

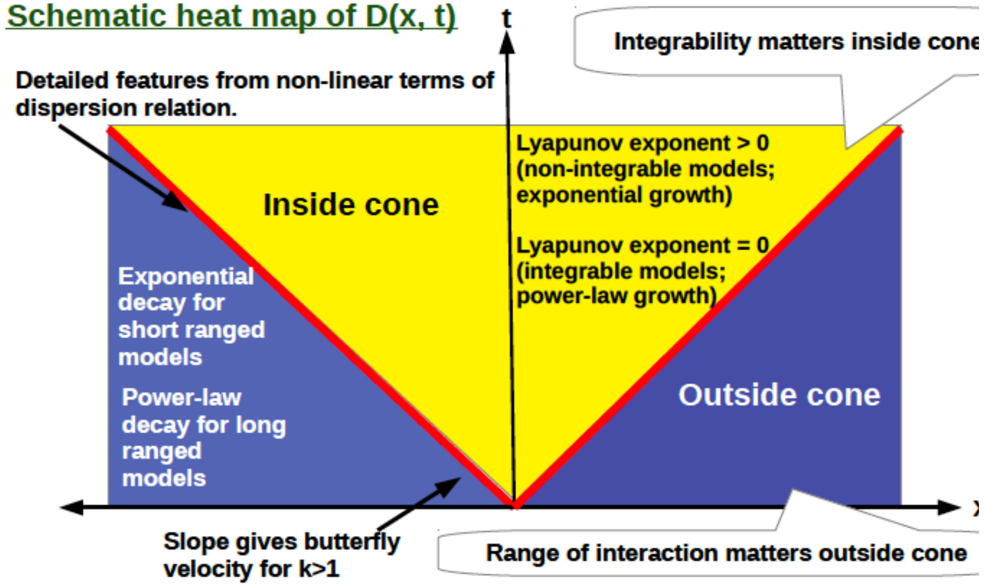


What to expect ?

OTOC Heat Map (Low Temperature)

B. Kiran, D. A. Huse, **MK** (PRE, 2021)

Schematic heat map of D(x, t)



Detailed features from non-linear terms of dispersion relation.

Integrability matters inside cone

Inside cone

Lyapunov exponent > 0 (non-integrable models; exponential growth)

Lyapunov exponent = 0 (integrable models; power-law growth)

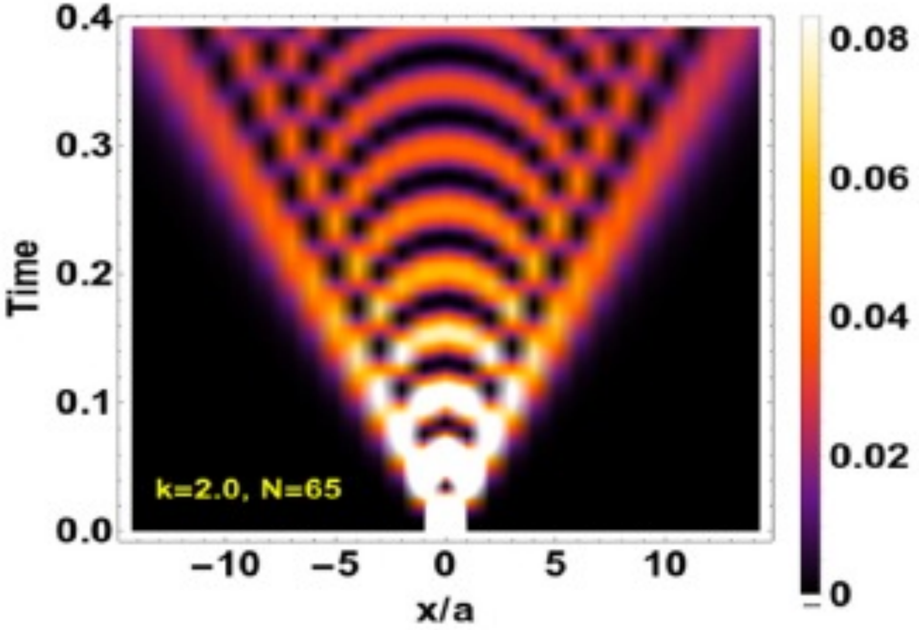
Exponential decay for short ranged models

Power-law decay for long ranged models

Outside cone

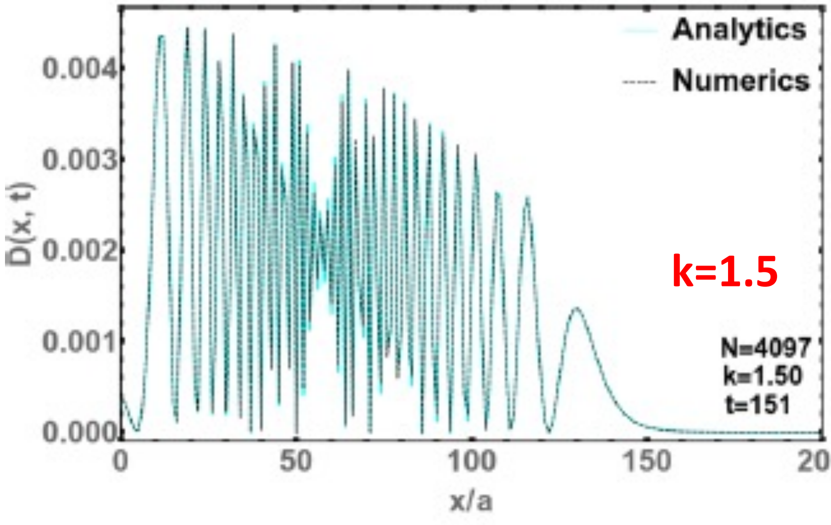
Slope gives butterfly velocity for k>1

Range of interaction matters outside cone

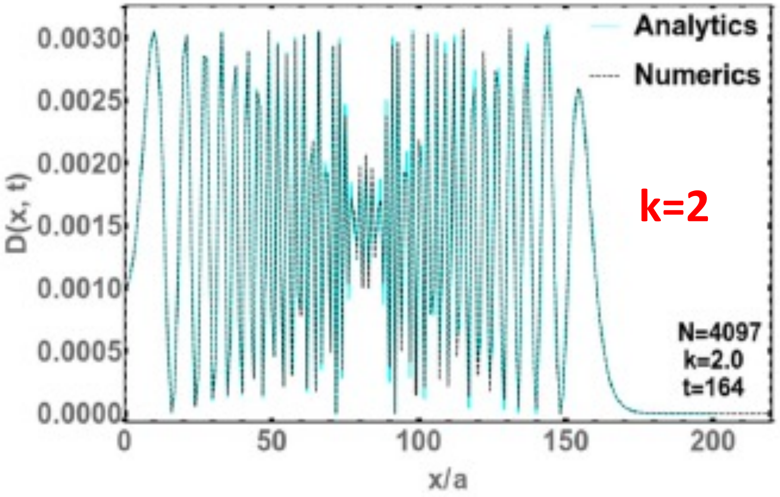


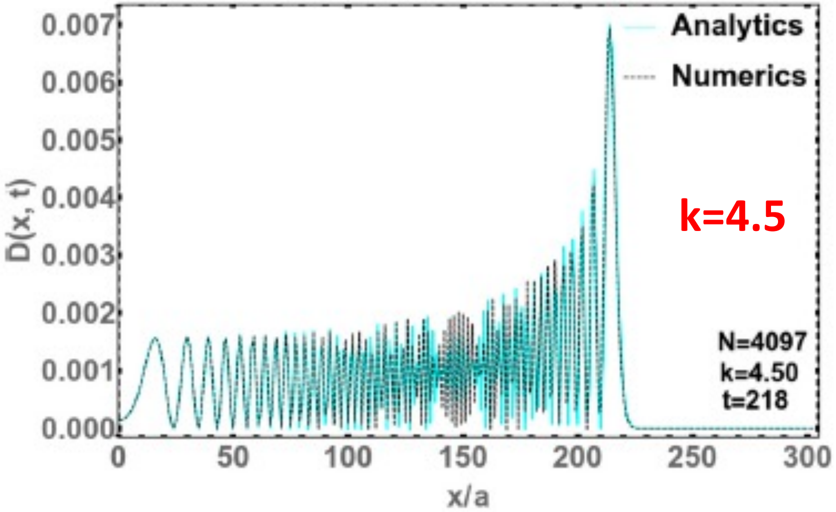
Key findings

- Exact analytical expressions for OTOC for integrable case
- Exact dispersion relation for all k > 1
- Heat map, butterfly velocity, asymptotics
- Also, a field theory approach



(a)





Conclusions

- Collective description and statistical properties of long ranged systems
- Results in presence of a wall and EVS
- Exact agreement between numerical and analytical results
- Bulk gap properties (mean and variance)
- Spacio-temporal spread of perturbations at low temperatures (OTOC) and other dynamical properties

- ➢ S. Agarwal, A. Dhar, **MK**, A. Kundu, S. N. Majumdar, D. Mukamel, G. Schehr, PRL (2019), Editors' Suggestion
- ➢ S. Agarwal, **MK**, A. Dhar, J. Stat Phys (2019)
- ➢ A. Kumar, **MK**, A. Kundu, PRE (2020)
- ➢ J. Kethepalli, **MK**, A. Kundu, S. N. Majumdar, D. Mukamel, G. Schehr J. Stat Mech (2021, 2022)
- ➢ S. Santra, J. Kethepalli, S. Agarwal, A. Dhar, **MK**, A. Kundu, S. N. Majumdar, G. Schehr, PRL (2022)
- ➢ B. Kiran, D. A. Huse, **MK** (PRE, 2021)

Outlook

- Full Counting Statistics (Number/Index, Kethepalli *et al*, In Preparation, 2023)
- Crossover from finite ranged to all-to-all coupling [Kumar, **M. K**, Kundu (2020), Santra, Kundu (2023)]
- Crossover between low temperature and high temperature
- Blast in Riesz gas (ongoing Mukherjee, Dhar, Ray, Krapivsky), Nonlinear Hydrodynamics
- Connection to experiments
- Recent interest in active systems (e.g., Gregory's talk, Touzo, Le Doussal, Schehr, EPL 2023, arXiv:2307.14306)

Thank you