Correlated Resetting Gas

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Ref: Phys. Rev. Lett. 130, 207101 (2023)

Longer version: arXiv: 2307.15351

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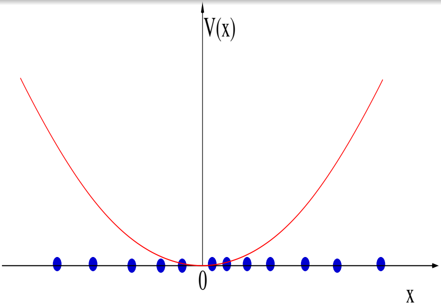
In preparation: M. Biroli, M. Kulkarni (ICTS, Bangalore), S.M., G. Schehr

Plan

- Correlated gas in thermal equilibrium: examples and observables
- Correlated gas in **nonequilibrium** stationary state created by **resetting**
- Exact results for various observables: average density, extreme and order statistics, gap statistics, full counting statistics
- Summary and Conclusion

One dimensional Correlated Gas
In
Thermal Equilibrium

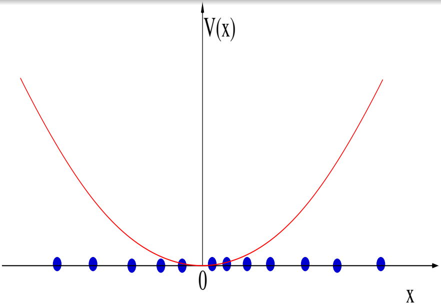
Correlated gas in thermal equilibrium



N particles on a line with coordinates
$\Longrightarrow \{x_1, x_2, \ldots, x_N\}$

$V(x) \rightarrow$ external confining potential

Correlated gas in thermal equilibrium



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Energy of the gas:

$$E[\{x_i\}] = \sum_i V(x_i) + \sum_{i \neq j} V_2(x_i, x_j) + \sum_{i \neq j \neq k} V_3(x_i, x_j, x_k) + \ldots$$

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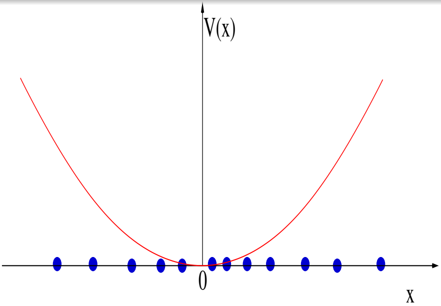

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$$P(x_1, x_2, \ldots, x_N) = \frac{1}{Z}\, e^{-\beta\, E[\{x_i\}]}$$

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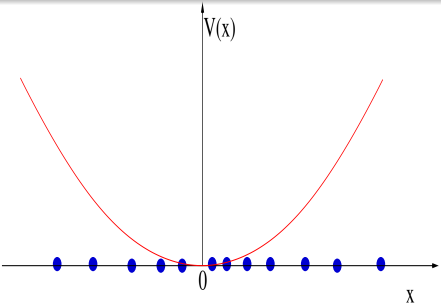

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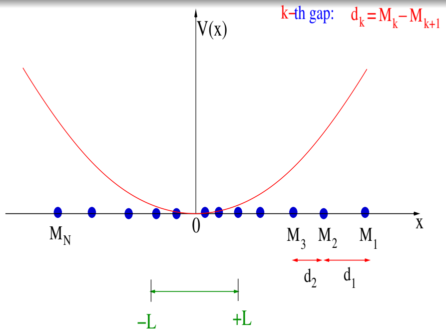
$$P(x_1, x_2, \ldots, x_N) = \frac{1}{Z} e^{-\beta\, E[\{x_i\}]} \neq p(x_1) p(x_2) \ldots p(x_N)$$

No **factorization** in the presence of **interactions**

Observables of interest



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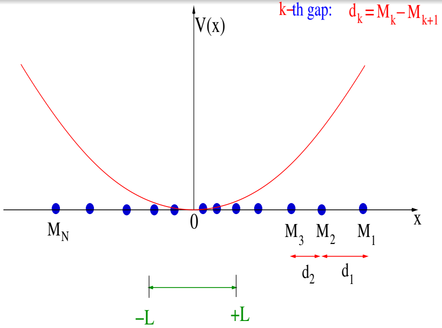

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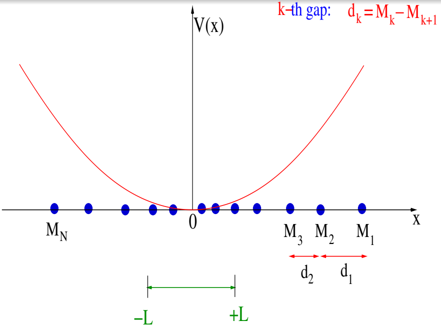

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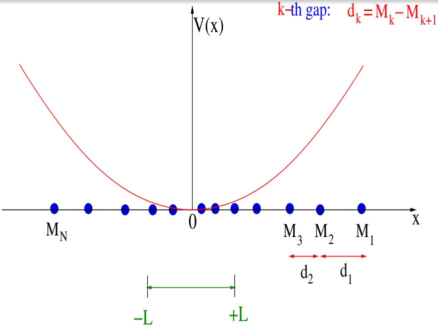

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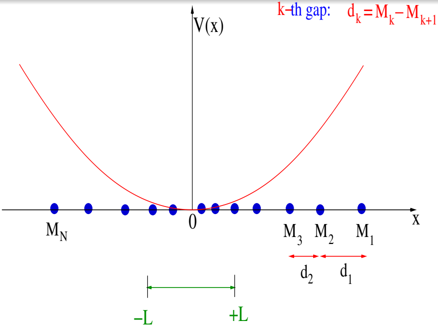

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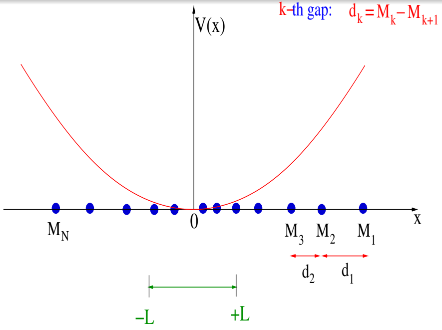

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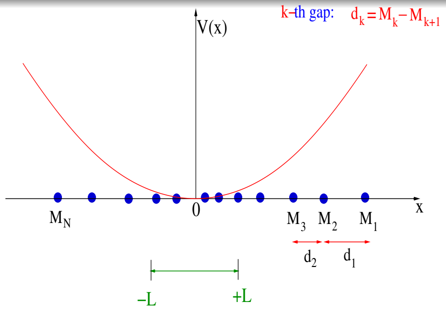
Generally hard to compute for a **correlated/interacting** gas !

Ideal gas: no interaction



In the absence of interactions

Energy: $E[\{x_i\}] = \sum_{i=1}^{N} V(x_i)$

Joint distribution **factorises** (i.i.d)

$P(\{x_i\}) = p(x_1)p(x_2)\dots p(x_N)$

where $p(x) = \frac{e^{-\beta\, V(x)}}{\int dx' e^{-\beta\, V(x')}}$

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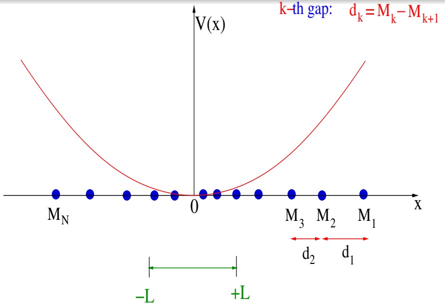

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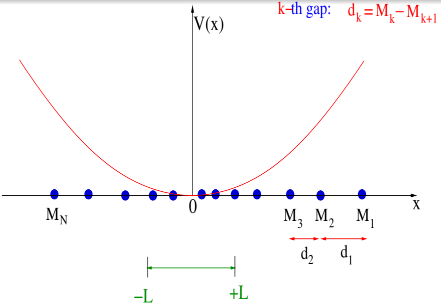

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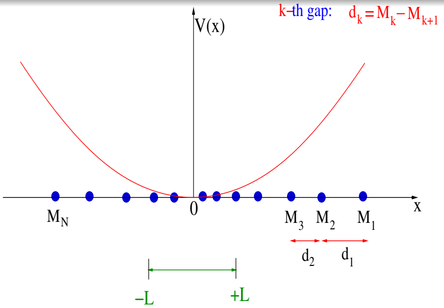

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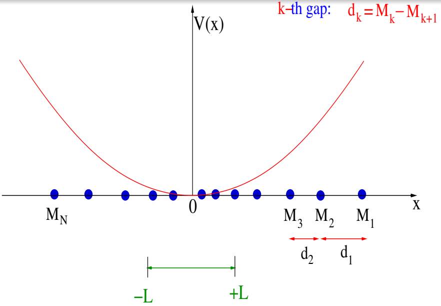

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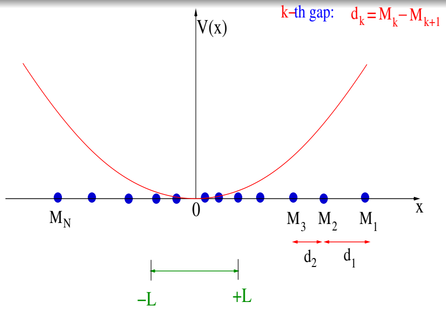

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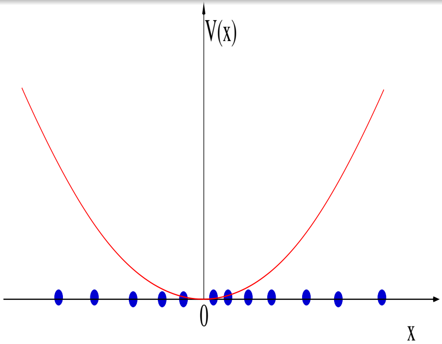
Example 1 of a correlated gas: Dyson's log-gas



Energy:

$$E[\{x_i\}] = \frac{N}{2}\sum_{i=1}^{N} x_i^2 - \frac{1}{2}\sum_{i\neq j} \log|x_i - x_j|$$

pairwise logarithmic repulsion Dyson, 1962

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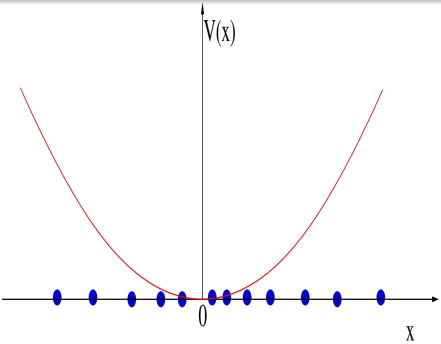

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Consider an $(N \times N)$ Gaussian Hermitian random matrix H_{ij} whose entries are distributed via:

$$\mathrm{Prob.}[H] \propto \exp\left[-N\sum_{i,j}|H_{ij}|^2\right] \propto \exp\left[-N\,\mathrm{Tr}\left(H^\dagger H\right)\right]$$

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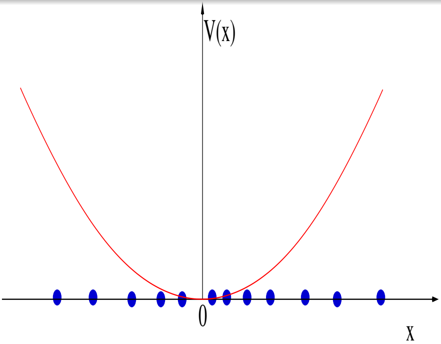

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N real eigenvalues: $\{\lambda_1, \lambda_2, \ldots, \lambda_N\} \longrightarrow$ **strongly correlated**

Dyson's log-gas

Joint distribution of eigenvalues of an $(N \times N)$ Gaussian Hermitian random matrix (Wigner, 1951):

$$P(\{\lambda_i\}) = \frac{1}{Z_N} \exp\left[-N \sum_{i=1}^{N} \lambda_i^2\right] \prod_{i<j} |\lambda_i - \lambda_j|^2$$

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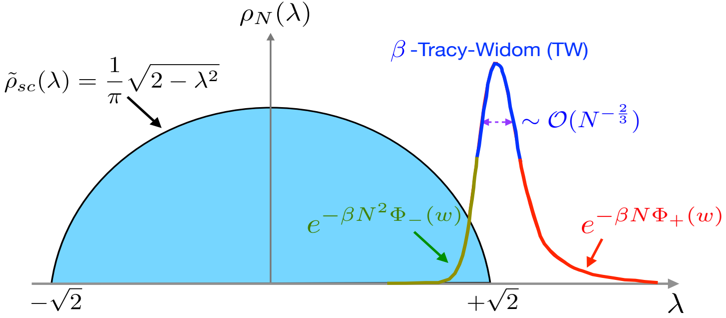
Hence one can identify the eigenvalues $\{\lambda_1, \lambda_2, \ldots \lambda_N\} \equiv \{x_1, x_2, \ldots, x_N\}$ as the positions of a 1-d gas of N particles with pairwise log-repulsion with $\beta = 2$ (Dyson, 1962)

Most of the observables can be computed exactly $\Longrightarrow$ not that **easy !**

Observables in the log-gas model



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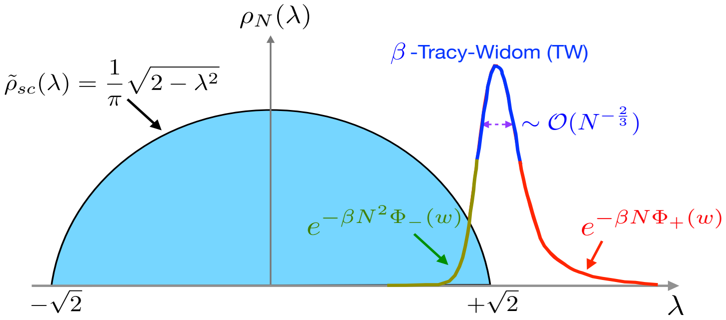

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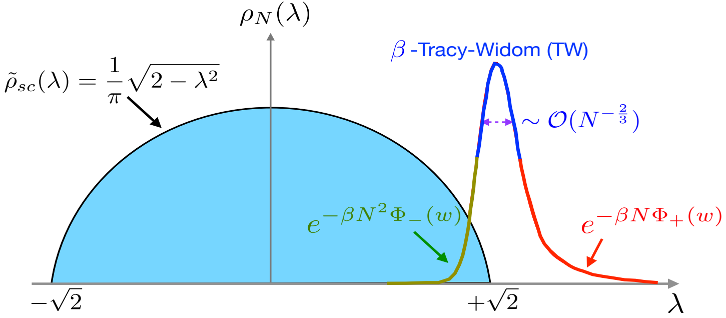

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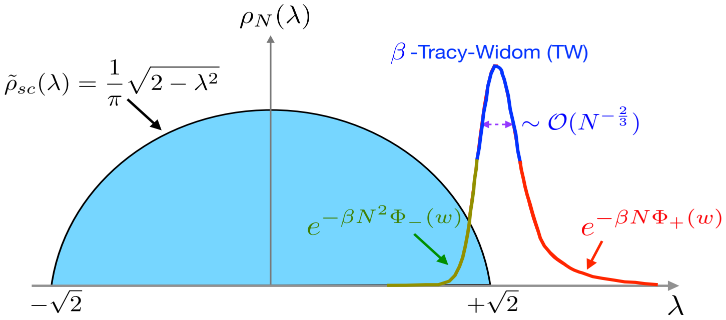

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Similarly, other observables are also known $\Longrightarrow$ huge literature

S.M., A. Pal, G. Schehr, "Extreme value statistics of correlated random variables: A pedagogical review", Phys. Rep. 840, 1 (2020).

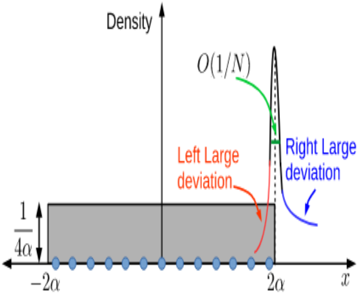
Ex 2: Jellium model in 1-d

Energy:

$$E[\{x_i\}] = \frac{N^2}{2}\sum_{i=1}^{N} x_i^2 - \alpha\, N \sum_{i\neq j} |x_i - x_j|$$

1-d Coulomb (linear) repulsion

Lenard, 1961; Prager, 1962; Baxter, 1963 ...



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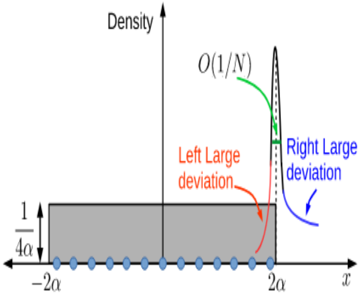

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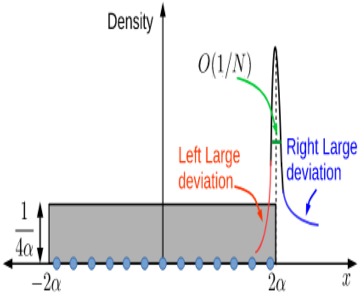

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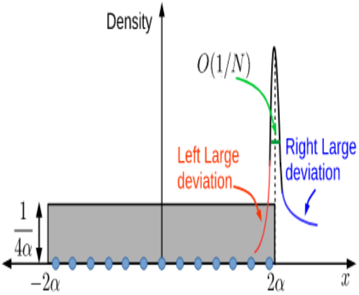

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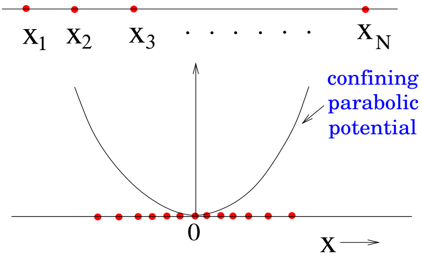
- Extreme, order, gap, full counting statistics $\Longrightarrow$ recently computed

Dhar, Kundu, S.M., Sabhapandit, Schehr, PRL, 119, 060601 (2017); J. Phys. A: Math. Theor. 51, 295001 (2018)

Flack, S.M., Schehr, J. Stat. Mech. 053211 (2022)

Ex 3: harmonically confined Riesz gas in 1-d



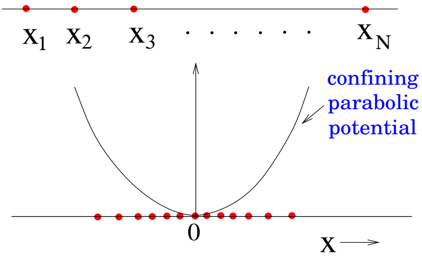
Energy function (with $k > -2$):

$$E[\{x_i\}] = \frac{1}{2}\sum_{i=1}^{N} x_i^2 + \frac{J\,\mathrm{sgn}(k)}{2}\sum_{i\neq j}\frac{1}{|x_i - x_j|^k}$$

M. Riesz, 1938

Recent survey: M. Lewin, 2022

Ex 3: harmonically confined Riesz gas in 1-d



Energy function (with $k > -2$):

$$E[\{x_i\}] = \frac{1}{2}\sum_{i=1}^{N} x_i^2 + \frac{J\,\mathrm{sgn}(k)}{2}\sum_{i\neq j}\frac{1}{|x_i - x_j|^k}$$

M. Riesz, 1938

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Special cases:

$k = -1$ (Jellium model), $k \to 0^+$ (Log-gas) and $k = 2$ (Calogero model)

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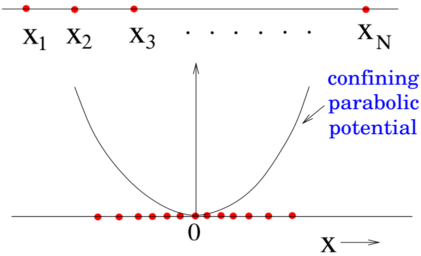

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$k = -1$ (Jellium model), $k \to 0^+$ (Log-gas) and $k = 2$ (Calogero model)

Average density $\rho(x, N)$ in the large N limit

$\Longrightarrow$ computed recently for all $k > -2$

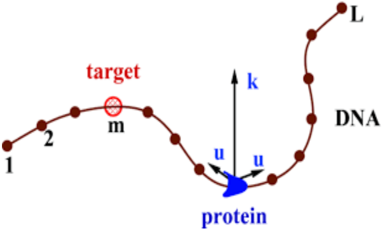
Agarwal, Dhar, Kulkarni, S.M., Mukamel, Schehr, PRL, 123, 100603 (2019)

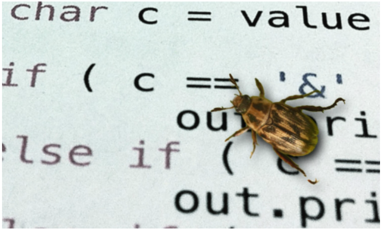
Kethepalli et. al., J. Stat. Mech., 103209 (2021); J. Stat. Mech. 033203 (2022)

Santra et. al. PRL, 128, 170603 (2022)

Nonequilibrium Stationary State
induced by
Stochastic Resetttting

Search problems are ubiquitous

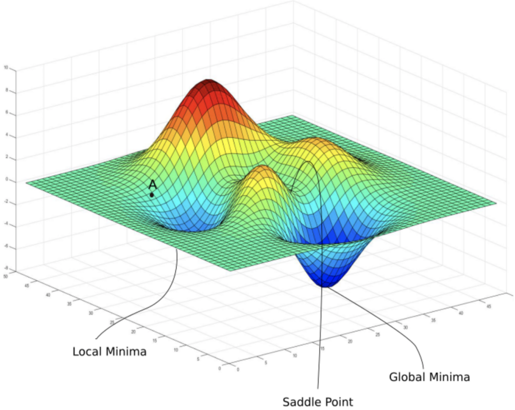




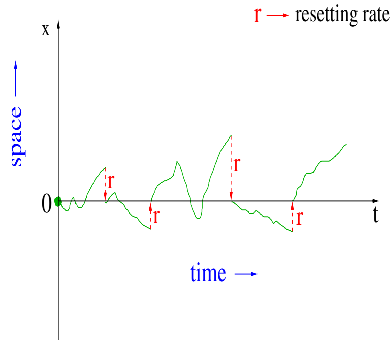
Other examples of stochastic resetting

- Searching for the global minimum in a complex energy landscape via simulated annealing

  empirical observation: Resetting to the initial configuration from time to time (and starting afresh) helps finding new pathways out of a metastable configuration



Diffusion with stochastic resetting: A simple model



Poissonian resetting

Time intervals between successive resettings distributed as:

$$p(\tau) = r\, e^{-r\tau}$$

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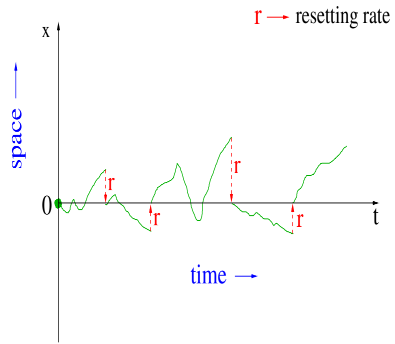

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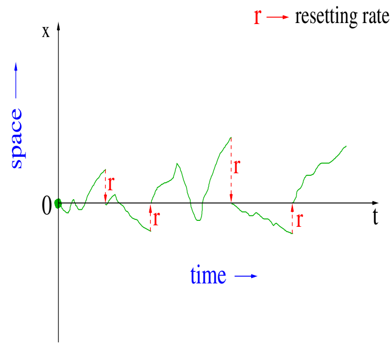
Dynamics: In a small time interval Δt

$$x(t+\Delta t) = 0 \qquad \text{with prob. } r\Delta t \qquad \text{(resetting)}$$

$$= x(t) + \eta(t)\,\Delta t \qquad \text{with prob. } 1 - r\Delta t \qquad \text{(diffusion)}$$



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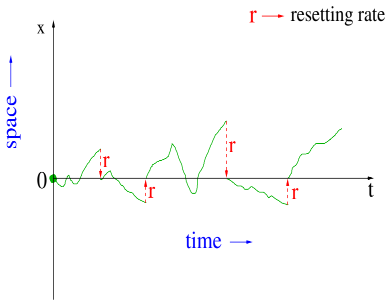
$\eta(t) \to$ Gaussian white noise: $\langle \eta(t) \rangle = 0$ and $\langle \eta(t)\eta(t') \rangle = 2\,D\,\delta(t-t')$

[M.R. Evans & S.M., PRL, 106, 160601 (2011)]

Prob. density $p_r(x,t)$ with resetting rate $r > 0$



$p_r(x,t) \to$ prob. density at time t,

given $p_r(x,0) = \delta(x)$

- In the absence of resetting ($r = 0$):

$$p_0(x,t) = \frac{1}{\sqrt{4\pi\, D\, t}}\, \exp[-x^2/4Dt]$$

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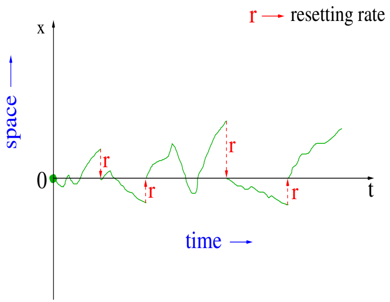

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$$p_r(x,t) = ?$$

Fokker-Planck (Master) Equation

Fokker-Planck Equation:

$$\partial_t p_r(x,t) = D\,\partial_x^2 p_r(x,t) - r\,p_r(x,t) + r\,\delta(x)\int p_r(x',t)dx'$$

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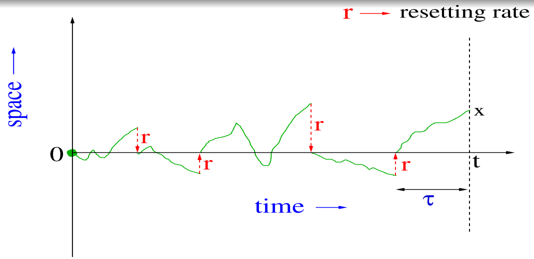
This linear equation can be solved at all t exactly by Fourier transform

Exact solution valid at all times t



- Exact solution at all times t:

$$p_r(x,t) = e^{-r\,t}\, p_0(x,t) + \int_0^t d\tau\, (r\, e^{-r\,\tau})\, p_0(x,\tau)$$

where $p_0(x,\tau) = \text{diffusion propagator} = \frac{1}{\sqrt{4\pi\, D\, \tau}}\, \exp[-x^2/4D\tau]$

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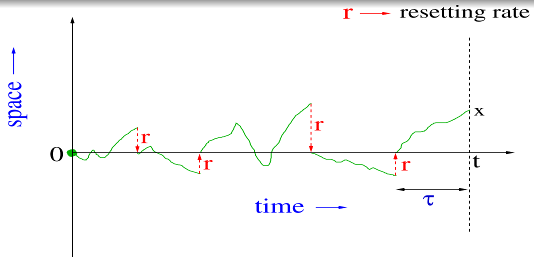

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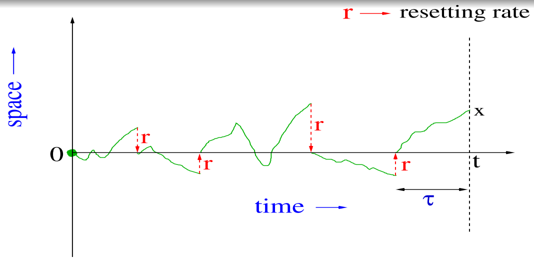

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- As $t \to \infty$, $p_r^{\rm st}(x) = r\, \int_0^\infty p_0(x,\tau)\, e^{-r\,\tau}\, d\tau = \frac{\alpha_0}{2}\, \exp[-\alpha_0\, |x|]$ where $\alpha_0 = \sqrt{r/D}$

Stationary State $\longrightarrow$ Nonequilibrium

Exact solution $\rightarrow$ $\boxed{p_r^{\rm st}(x) = \frac{\alpha_0}{2}\, \exp[-\alpha_0\, |x|]}$ with $\alpha_0 = \sqrt{r/D}$

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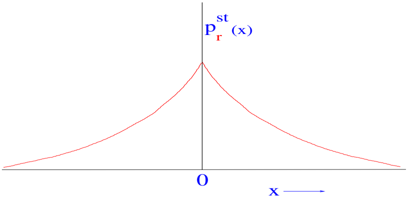

$\rightarrow$ nonequilibrium stationary state (NESS)

$\Rightarrow$ current carrying with detailed balance $\rightarrow$ violated

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effective potential: $\alpha_0 |x|$

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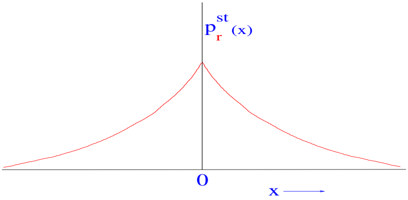

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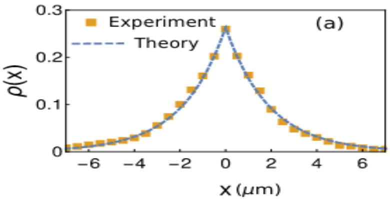
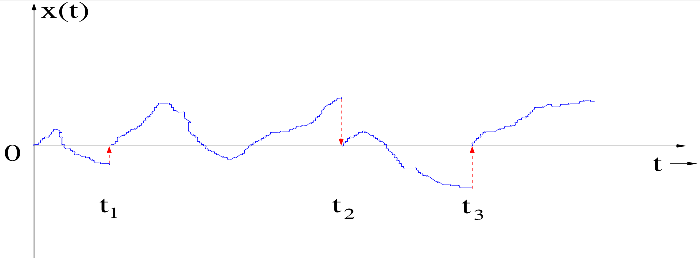


Recent experiment using holographic optical tweezers

Tal-Friedman, Pal, Sekhon, Reuveni, & Roichman
J. Phys. Chem. Lett. 11, 7350 (2020)

Generalisations to many-body systems

Stochastic Resetting in a nutshell



- Consider any process $x(t)$ evolving freely by its own dynamics (deterministic or stochastic) during a certain random interval of time
- At the end of this random period, the process is reset to its initial position (or some randomly chosen position) and the its dynamics restarts afresh
- The interval of free evolution between resets is drawn independently from a distribution $p(\tau) \Longrightarrow$ renewal process
- For Poissonian resetting with a constant rate r: $p(\tau) = r\, e^{-r\tau}$

Generalisation to many-body system

Consider any many-body system (with interaction) evolving under its own stochastic dynamics

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Ex: fluctuating interfaces, Ising model with Glauber dynamics etc.

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Configuration C: $\{H_1, H_2, \ldots, H_L\} \rightarrow$ heights of an $(1+1)$-dim KPZ/EW interface

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Renewal equation: Setting $\tau \to$ time since last resetting before t

$$P_r(C,t) = e^{-r\,t}\, P_0(C,t) + \int_0^t d\tau\, (r\, e^{-r\,\tau})\, P_0(C,\tau)$$

[S. Gupta, S.M., G. Schehr, PRL, 112, 220601 (2014)]

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As $t \to \infty$, the nonequilibrium stationary state:

$$P_r^{\rm st}(C) = \int_0^\infty d\tau\, (r\, e^{-r\,\tau})\, P_0(C,\tau)$$

Correlated Resetting Gas

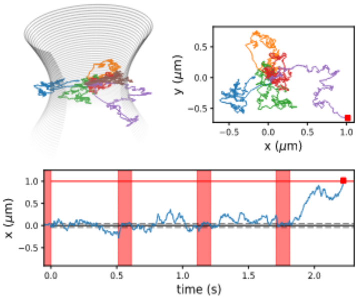
Recent Experiments on Stochastic Resetting

Recent experiments on stochastic resetting using optical traps set-up:

Tal-Friedman, Pal, Sekhon, Reuveni, Roichman, J. Phys. Chem. Lett. 11, 7350 (2020)

Besga, Bovon, Petrosyan, S.M., Ciliberto, Phys. Rev. Res. 2, 032029 (2020) $\longrightarrow$ 1-dimension

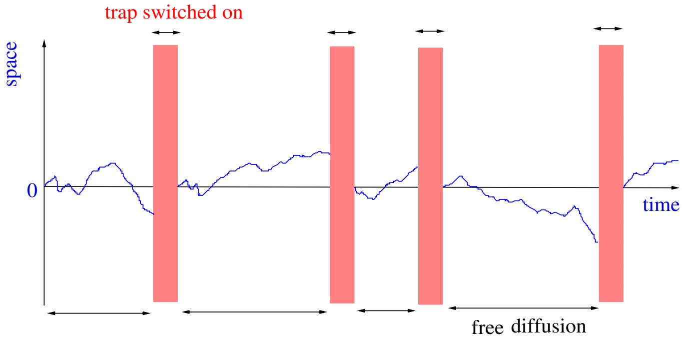
Faisant, Besga, Petrosyan, Ciliberto, S.M. J. Stat. Mech. 113203 (2021) $\longrightarrow$ 2-dimension



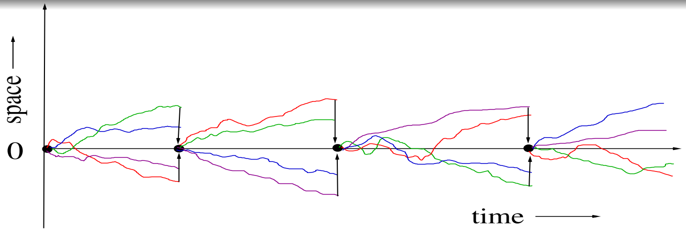
1. Free diffusion for a certain period (deterministic or random)

2. Switch on an optical harmonic trap and the let the particle relax to its equilibrium distribution using Enginnered Swift Equilibration (ESE) technique $\Longrightarrow$ mimics instantaneous resetting

Steps 1 and 2 alternate ...

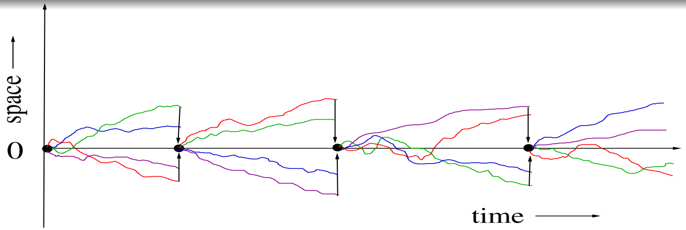


Correlated resetting gas



Consider N Brownian motions (independent) that are **simultaneously** reset with rate r to the origin

Correlated resetting gas



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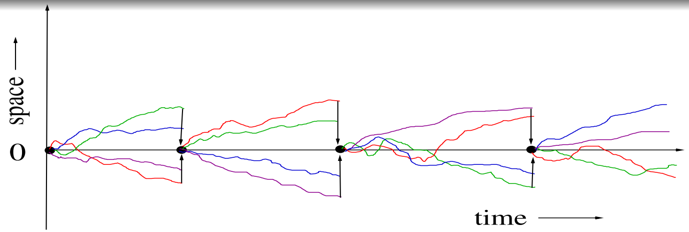
The joint position distribution approaches a nonequilibrium stationary state (NESS) at long times

Correlated resetting gas



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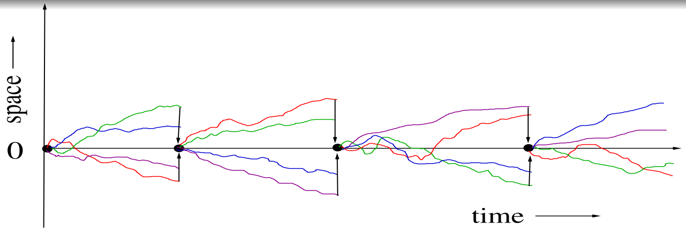

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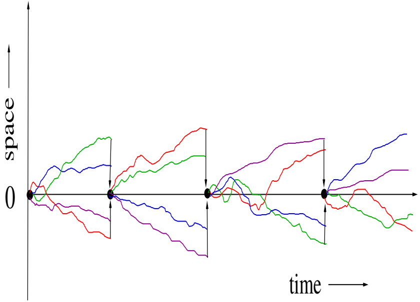
The joint distribution does not **factorize** $\Longrightarrow$ **correlated** resetting gas

M. Biroli, H. Larralde, S. M., G. Schehr, PRL, 130, 207101 (2023)

Solvable Correlated Gas



Joint distribution:

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$$p_0(x,\tau) = \frac{1}{\sqrt{4\pi D\tau}}\, e^{-x_i^2/4D\tau}$$

In this model, interactions between particles are not built-in, but the correlations are generated by the dynamics (**simultaneous resetting**), that persist all the way to the stationary state

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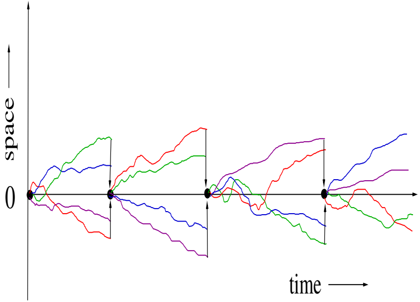

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The gas is **strongly** correlated in the NESS

$$\langle x_i^2 x_j^2 \rangle - \langle x_i^2 \rangle \langle x_j^2 \rangle = 4\frac{D^2}{r^2} \Longrightarrow$$ **attractive all-to-all** interaction

Solvable Correlated Gas

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Despite **strong correlations**, several physical observables can be computed **exactly** in the NESS $\Longrightarrow$ (Solvable)

- Compute any observable for the ideal gas $\Rightarrow$ I.I.D variables with distribution $p_0(x,\tau)$ parametrized by $\tau \Longrightarrow$ easy
- Average over the **random** parameter τ using $p(\tau) = r\, e^{-r\,\tau}$

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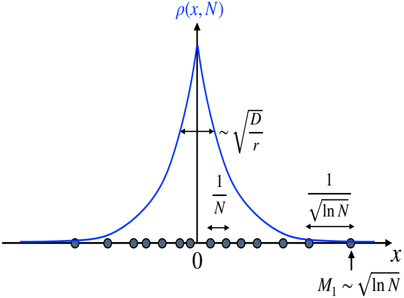
Examples:

- Average density
- Distribution of the k-th maximum: Order statistics
- Spacing distribution
- Full Counting Statistics

M. Biroli, H. Larralde, S. M., G. Schehr, PRL, 130, 207101 (2023)

Average Density



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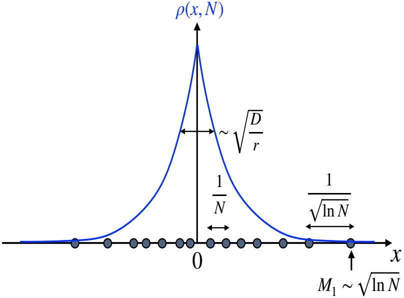
Average density:

$$\rho(x,N) = \frac{1}{N}\sum_{i=1}^{N}\langle\delta(x_i - x)\rangle = \int P_r^{\rm st}\left(x,\, x_2,\ldots,x_N\right)\, dx_2\, dx_3\ldots dx_N$$

Average Density



Joint distribution:

$$P_r^{\rm st}(\{x_i\}) = r\int_0^\infty d\tau\, e^{-r\tau}\prod_{i=1}^{N} p_0(x_i,\tau)$$

$$p_0(x,\tau) = \frac{1}{\sqrt{4\pi D\tau}}\, e^{-x_i^2/4D\tau}$$

Average density:

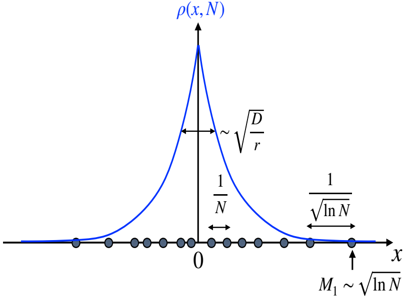
$$\rho(x,N) = \frac{1}{N}\sum_{i=1}^{N}\langle\delta(x_i-x)\rangle = \int P_r^{\rm st}(x,\, x_2,\ldots,x_N)\; dx_2\, dx_3\ldots dx_N$$

$$= r\int_0^\infty d\tau\, e^{-r\tau}\, p_0(x,\tau) = \frac{\alpha_0}{2}\,\exp[-\alpha_0\,|x|]$$

where $\alpha_0 = \sqrt{r/D}$

$\Longrightarrow$ same as the single particle position distribution

Order Statistics



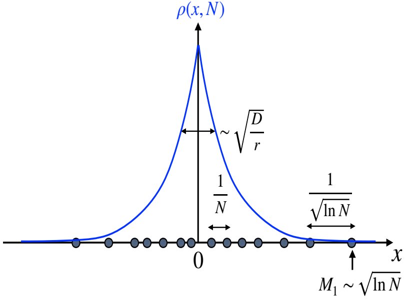
$M_k \Longrightarrow k$-th maximum

Set $k = \alpha\, N$

$\alpha \sim O(1) \Longrightarrow$ **bulk**

$\alpha \sim O(1/N) \Longrightarrow$ **edge**

Order Statistics



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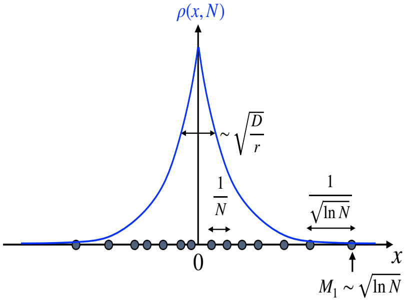
Order Statistics



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- Edge: $\mathrm{Prob.}[M_k = w] \approx \frac{1}{L_N} f\left(\frac{w}{L_N}\right)$ where $L_N = \sqrt{\frac{4D\,\ln N}{r}}$

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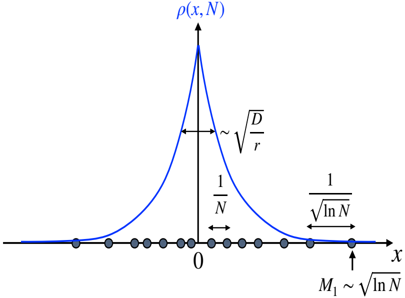

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The scaling function $\mathbf{f(z) = 2\,z\,e^{-z^2}\,\theta(z)} \Longrightarrow$ **universal** (indep. of α)

M. Biroli, H. Larralde, S. M., G. Schehr, PRL, 130, 207101 (2023)

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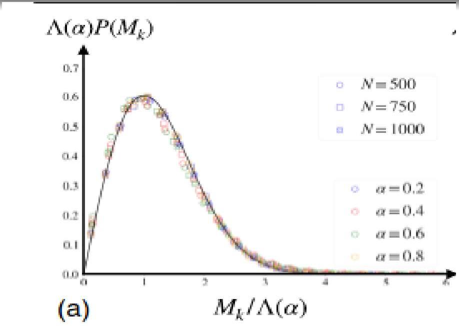

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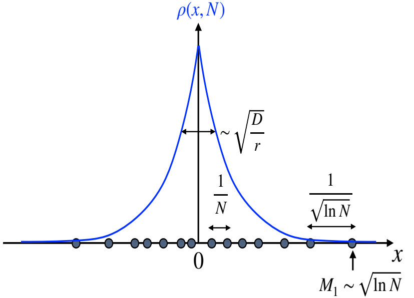
Gap/Spacing Statistics



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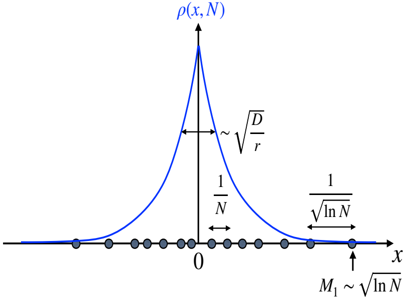

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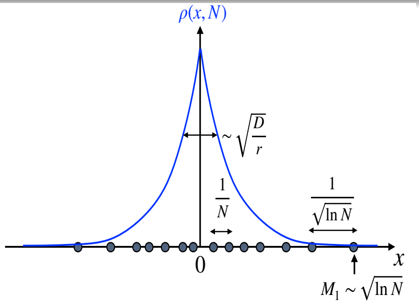
Gap/Spacing Statistics



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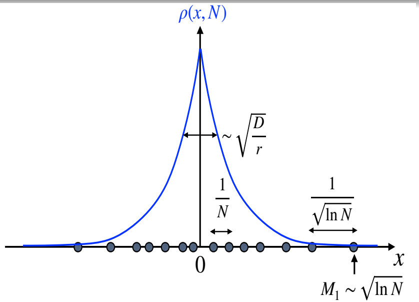

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The scaling function $h(z) = 2\int_0^\infty du\, e^{-u^2 - z/u}$ $\quad (z \geq 0)$

$\Longrightarrow$ **universal** (indep. of α)

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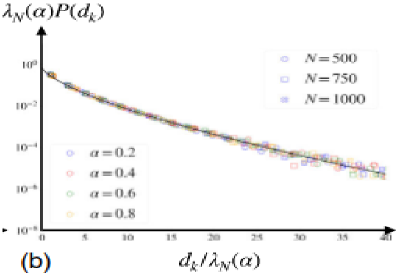

The gap scaling function:

$h(z) = 2\int_0^\infty du\, e^{-u^2 - z/u}$

$h(z) \to \sqrt{\pi}$ as $z \to 0$

$h(z) \sim \exp[-3(z/2)^{2/3}]$ as $z \to \infty$

- Bulk: $\mathrm{Prob.}[d_k = g] \approx \frac{1}{\lambda_N(\alpha)}\, h\left(\frac{g}{\lambda_N(\alpha)}\right)$ where $\lambda_N(\alpha) = \frac{1}{b\sqrt{r}\,N}$ with $b = \exp\left(-[\mathrm{erfc}^{-1}(2\,\alpha)]^2\right)/\sqrt{4\pi D}$
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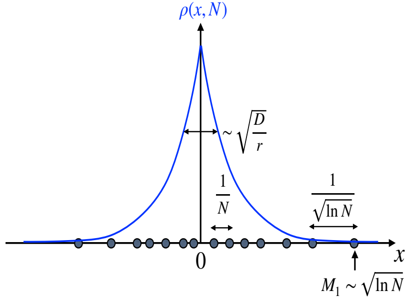
Full Counting Statistics

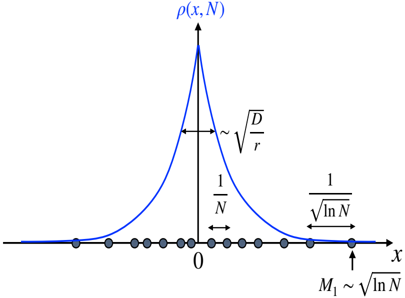


$N_L \Longrightarrow$ number of particles in $[-L, L]$

Clearly, $0 \leq N_L \leq N$

$P(N_L, N) = ?$

Full Counting Statistics



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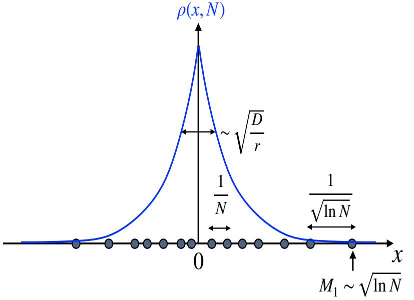

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where the scaling function:

$$H(\kappa) = \gamma\sqrt{\pi}\,[u(\kappa)]^{-3}\, \exp\left[-\gamma\, u^{-2}(\kappa) + u^2(\kappa)\right]$$

with $\gamma = r\, L^2/(4D)$ and $u(\kappa) = \mathrm{erf}^{-1}(\kappa)$

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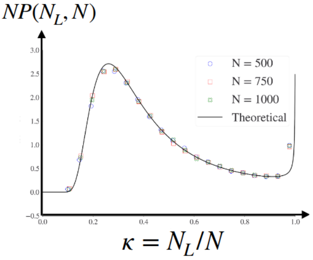

The scaling function $H(\kappa)$

$H(\kappa) \to \frac{8\gamma}{\pi\,\kappa^3}\ \exp\left[-\frac{4\gamma}{\pi\,\kappa^2}\right]$ as $\kappa \to 0$

$H(\kappa) \to \frac{\gamma\,\sqrt{\pi}}{(1-\kappa)\,[\ln(1-\kappa)]^{3/2}}$ as $\kappa \to 1$

Full Counting Statistics: $P(N_L, N) \approx \frac{1}{N}\, H\left(\frac{N_L}{N} = \kappa\right) \quad (0 \le \kappa \le 1)$

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M. Biroli, H. Larralde, S. M., G. Schehr, PRL, 130, 207101 (2023)

Generalisations

The structure of the joint distribution for N independent particles driven by simultaneous resetting is very general:

$$P_r^{\rm st}\left(\{x_i\}\right) = r \int_0^\infty d\tau\, e^{-r\,\tau} \prod_{i=1}^{N} p_0(x_i, \tau)$$

where $p_0(x,\tau)$ can represent **any** single particle motion, not necessarily difusion

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where $p_0(x,\tau)$ can represent **any** single particle motion, not necessarily difusion

Ex: ballistic motion, Lévy flights etc.

$\Longrightarrow$ a whole class of **solvable** correlated gases in their nonequilibrium stationary state

$\Longrightarrow$ a new application of stochastic resetting

M. Biroli, H. Larralde, S. M., G. Schehr, arXiv: 2307.15351

Summary and Conclusions

- A simple solvable model of a correlated gas of N diffusing particles in their nonequilibrium stationary state driven by simultaneous stochastic resetting

- Several physical observables are exactly computable and have rich and interesting behaviors, despite being a **strongly correlated** system

- Easily generalisable to a whole new class of solvable correlated gases in their nonequilibrium stationary state $\longrightarrow$ ballistic particles, Lévy flights

- Generalisation to N independent particles in a confining harmonic potential that swtiches between two stiffnesses μ_1 and μ_2 with rate r

M. Biroli, M. Kulkarni, S.M., G. Schehr (in preparation)

Other applications of stochastic resetting

- Unusual temporal relaxation to the stationary state
- Target search with optimal resetting rate
- Enzymatic reactions in biology (Michaelis-Menten reaction)
- Lévy flights, Lévy walks, fractional BM with resetting
- Space-time dependent resetting rate
- Search via nonequilibrium reset dynamics vs. equilibrium dynamics
- Resetting dynamics of extended systems (Ising model, interfaces)
- Memory dependent reset
- First-passage resetting
- Quantum dynamics with reset
- Active particles with reset
- Territory covered by resetting Brownian motions in 2-d
- Resetting Brownian motion with constraints (Br. bridge)
- Cost of resetting
- Stochastic optimal control theory
- Queuing theory ... $\Longrightarrow$ a long list **!**

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Recent reviews on Stochastic Resetting and Applications:

M.R. Evans, S.M., G. Schehr, J. Phys. A. : Math. Theor. 53, 193001 (2020)

A. Pal, S. Kostinki, S. Reuveni, J. Phys. A: Math. Theor. 55, 021001 (2022)

S. Gupta, A.M. Jayannavar, Front. Phys. 10, 789097 (2022)

Journal of Physics A: Mathematical and Theoretical

Stochastic Resetting: Theory and Applications

In Celebration of the 10th Anniversary of 'Diffusion with Stochastic Resetting'

Guest Editors

Anupam Kundu *International Centre for Theoretical Sciences, Bengaluru, India*
Shlomi Reuveni *Tel Aviv University, Israel*

Scope

Restart is a simple and natural mechanism that has emerged as an overreaching topic in physics, chemistry, biology, ecology, engineering and economics. Since the inaugural work of Evans and Majumdar (Evans M R and Majumdar S N 2011, *Phys. Rev. Lett.* **106**, 160601) a substantial amount of research has been carried out on stochastic resetting and its applications. This work spans different contexts starting from first-passage and search theory, stochastic thermodynamics, optimization theory, and all the way to quantum mechanics. Further connections have been made to animal foraging, protein-DNA interactions, coagulation-diffusion processes, chemical reaction processes, as well as to stock-market and population dynamics which display colossal crashes, i.e., resetting events. While most studies to date have been theoretical, several experimental groups have now entered the playing field, which marks the dawn of a new era.

The goal of this issue is to report new and original advancements made on stochastic resetting and applications, to open novel research directions and to attract additional researchers to work in the exciting field of stochastic resetting.

Stochastic Resetting $\rightarrow$ rich and interesting static/dynamic phenomena

Collaborators on Stochastic Resetting

- M. Biroli, I. Burenev, B. De Bruyne, C. Di Bello, M. Gueneau, M. Magoni, F. Mori (Master & Ph. D students, LPTMS & LPTHE); F. Mori ($\Rightarrow$ Oxford, UK)
- D. Boyer, H. Larralde, A. Falcon-Cortes, G. Marcado-Vasquez (UNAM, Mexico)
- S. Ciliberto & group (ENS-Lyon, France)
- F. den Hollander (Leiden University, The Netherlands)
- M. R. Evans, J. Whitehouse (Edinburgh University, UK), L. Giuggioli (Bristol, UK)
- A. Kundu, M. Kulkarni (ICTS, Bangalore), S. Gupta (TIFR, Bombay)
- A. K. Hartmann (Oldenburg Univ,, Germany)
- L. Kusmierz (Inst. of Phys., Krakow, Poland $\rightarrow$ Riken Center, Japan)
- K. Mallick (IPHT, Saclay)
- J. M. Meylahn, H. Touchette (Stellenbosch University, South Africa)
- B. Mukherjee, K. Sengupta (IACS, Kolkata, India)
- G. Oshanin (LPTMC, Paris)
- A. Rosso (LPTMS, Orsay)
- S. Sabhapandit (RRI, Bangalore, India)
- H. Schawe (Cergy-Pontoise, France)
- G. Schehr (LPTHE, Sorbonne University, France)
- N. R. Smith (Ben-Gurion Univ., Israel)

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