Thermodynamic costs for resetting processes

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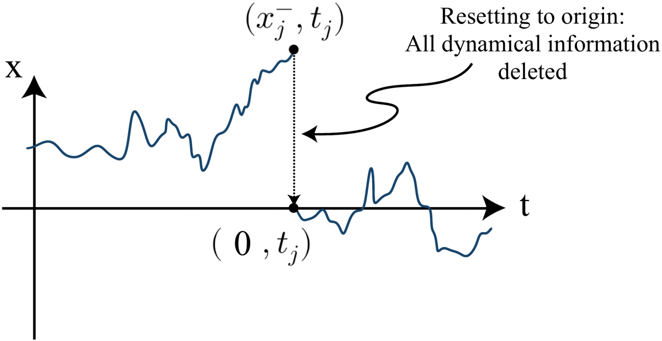
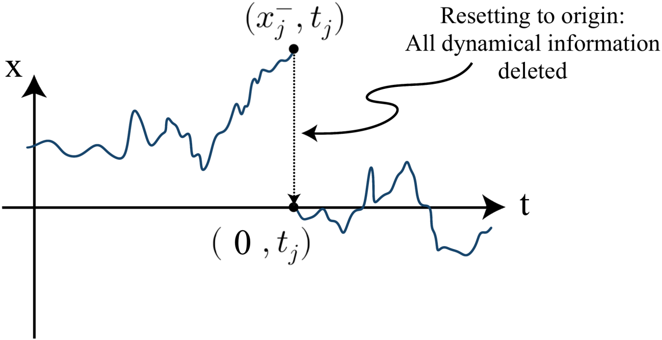
Resetting



[1]

$$x(\tau + dt) = \begin{cases} x(\tau) - V'(x)dt + \sqrt{2D}\eta(\tau)dt & \text{with probability } 1 - r\,dt\,, \\ x_{\rm res} & \text{with probability } r\,dt\,, \end{cases}$$

[1] Evans and Majumdar, "Diffusion with stochastic resetting".



[2]

$$\partial_t p(x,t) = -\partial_x j(x,t) - rp(x,t) + r\delta(x)\,,$$

where

$$j(x,t) = -D\partial_x p(x,t) - V'(x)p(x,t)$$

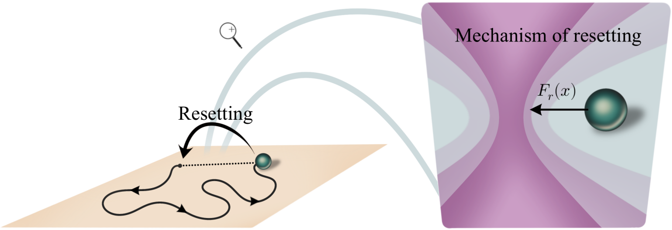
In steady state

$$j_{\rm st}(x) = -D\partial_x p_{\rm st}(x) - V'(x)p_{\rm st}(x)\,.$$

[2] Evans and Majumdar, "Diffusion with stochastic resetting".

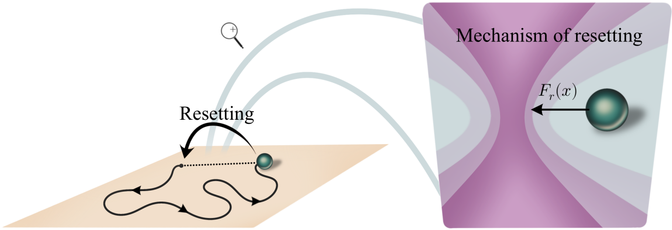
Thermodynamic cost for Resetting

A cost for a reset can be calculated as the **work** for implementing a reset by some mechanism that does the actual resetting



Thermodynamic cost for Resetting

Or as the **heat dissipated or entropy produced** by the resetting process. (Resetting results in a NESS and a NESS generates entropy). Cost= Total entropy production



Thermodynamic costs[3] for processes described by overdamped Langevin equations (with conservative forces)

$$\frac{dx_\tau}{d\tau} = -\gamma^{-1}\nabla V_{\text{tot}}[x_\tau, \Lambda(\tau)] + \eta(\tau)\,,$$

$$w = \int_0^t \frac{\partial V_{\text{tot}}}{\partial \Lambda}\frac{d\Lambda}{dt'}dt'$$

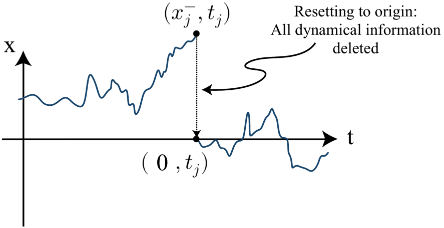
$$q = \int_0^t \frac{\partial V_{\text{tot}}}{\partial x} \circ dx = \log \frac{\mathcal{P}\left(\{x_\tau\}_0^\tau | x_0\right)}{\hat{\mathcal{P}}\left(\{\hat{x}_\tau\}_0^\tau | \hat{x}_0\right)}$$

$$S_{\text{tot}} = \log \frac{\mathcal{P}\left(\{x_\tau\}_0^\tau\right)}{\hat{\mathcal{P}}\left(\{\hat{x}_\tau\}_0^\tau\right)} = q + S_{\text{sys}}$$

$$\langle \dot{S}_{\text{tot}} \rangle = \frac{1}{D}\int_{-\infty}^{\infty} dx\, \frac{j_{\text{st}}^2(x)}{p_{\text{st}}(x)}\,.$$

[3] Sekimoto, *Stochastic energetics*; Seifert, "Stochastic thermodynamics, fluctuation theorems and molecular machines".

Perfect resetting "cost"[4]



$$\partial_t p(x,t) = -\partial_x j(x,t) - rp(x,t) + r\delta(x)$$

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In steady state

$$j_{\mathrm{st}}(x) = -D\partial_x p_{\mathrm{st}}(x) - V'(x)p_{\mathrm{st}}(x)$$

$$\langle \dot{S}_{\mathrm{tot}} \rangle \to \frac{1}{D}\int_{-\infty}^{\infty} dx\, \frac{j_{\mathrm{st}}^2(x)}{p_{\mathrm{st}}(x)}$$

[4] Fuchs, Goldt, and Seifert, "Stochastic thermodynamics of resetting".

Unidirectional transitions "cost"[5]: a brief diversion

$$\frac{dp_i}{dt} = \sum_{j=1}^{N} \left(w_{j\to i} p_j - w_{i\to j} p_i \right) + \sum_{j=1}^{N} \left(y_{j\to i} p_j - y_{i\to j} p_i \right) \, .$$

[5] Busiello, Gupta, and Maritan, "Entropy production in systems with unidirectional transitions"; Busiello, Hidalgo, and Maritan, "Entropy production for coarse-grained dynamics".

Unidirectional transitions "cost"[5]: a brief diversion

$$\frac{dp_i}{dt} = \sum_{j=1}^{N} \left(w_{j\to i} p_j - w_{i\to j} p_i \right) + \sum_{j=1}^{N} \left(y_{j\to i} p_j - y_{i\to j} p_i \right) \; .$$

$$\langle S_{\text{sys}} \rangle = - \sum_i p_i \log p_i$$

[5] Busiello, Gupta, and Maritan, "Entropy production in systems with unidirectional transitions"; Busiello, Hidalgo, and Maritan, "Entropy production for coarse-grained dynamics".

Unidirectional transitions "cost"[5]: a brief diversion

$$\frac{dp_i}{dt} = \sum_{j=1}^{N} \left(w_{j\to i} p_j - w_{i\to j} p_i \right) + \sum_{j=1}^{N} \left(y_{j\to i} p_j - y_{i\to j} p_i \right) .$$

$$\langle S_{\text{sys}} \rangle = -\sum_i p_i \log p_i$$

$$\langle \dot{S_{\text{sys}}}(t) \rangle = \sum_{i,j} w_{j\to i} p_j \log \frac{p_j}{p_i} + \sum_{i,j} y_{j\to i} p_j \log \frac{p_j}{p_i}$$

[5] Busiello, Gupta, and Maritan, "Entropy production in systems with unidirectional transitions"; Busiello, Hidalgo, and Maritan, "Entropy production for coarse-grained dynamics".

Unidirectional transitions "cost"[5]: a brief diversion

$$\frac{dp_i}{dt} = \sum_{j=1}^{N} (w_{j\to i} p_j - w_{i\to j} p_i) + \sum_{j=1}^{N} (y_{j\to i} p_j - y_{i\to j} p_i) \ .$$

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$$\langle \dot{S_{\text{sys}}}(t) \rangle = \sum_{i,j} w_{j\to i} p_j \log \frac{p_j}{p_i} + \sum_{i,j} y_{j\to i} p_j \log \frac{p_j}{p_i}$$

$$\langle \dot{S_{\text{sys}}}(t) \rangle = \sum_{i,j} w_{j\to i} p_j \log \frac{w_{j\to i} p_j}{w_{i\to j} p_i} - \sum_{i,j} w_{j\to i} p_j \log \frac{w_{j\to i}}{w_{i\to j}} + \sum_{i,j} y_{j\to i} p_j \log \frac{p_j}{p_i}$$

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Unidirectional transitions "cost"[5]: a brief diversion

$$\frac{dp_i}{dt} = \sum_{j=1}^{N} (w_{j\to i}p_j - w_{i\to j}p_i) + \sum_{j=1}^{N} (y_{j\to i}p_j - y_{i\to j}p_i) \; .$$

$$\langle S_{\text{sys}} \rangle = -\sum_i p_i \log p_i$$

$$\langle \dot{S_{\text{sys}}}(t) \rangle = \sum_{i,j} w_{j\to i}p_j \log \frac{p_j}{p_i} + \sum_{i,j} y_{j\to i}p_j \log \frac{p_j}{p_i}$$

$$\langle \dot{S_{\text{sys}}}(t) \rangle = \sum_{i,j} w_{j\to i}p_j \log \frac{w_{j\to i}p_j}{w_{i\to j}p_i} - \sum_{i,j} w_{j\to i}p_j \log \frac{w_{j\to i}}{w_{i\to j}} + \sum_{i,j} y_{j\to i}p_j \log \frac{p_j}{p_i}$$

In steady state,

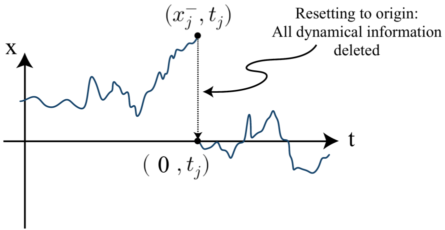
$$\sum_{i,j} w_{j\to i}p_j^s \log \frac{w_{j\to i}p_j^s}{w_{i\to j}p_i^s} = \sum_{i,j} w_{j\to i}p_j^s \log \frac{w_{j\to i}}{w_{i\to j}} - \sum_{i,j} y_{j\to i}p_j^s \log \frac{p_j^s}{p_i^s}$$

If y transitions were bi-directional,

$$\langle \dot{S}_{\text{tot}} \rangle = \sum_{i,j} w_{j\to i}p_j^s \log \frac{w_{j\to i}p_j^s}{w_{i\to j}p_i^s} + \sum_{i,j} y_{j\to i}p_j^s \log \frac{y_{j\to i}p_j^s}{y_{i\to j}p_i^s}$$

[5] Busiello, Gupta, and Maritan, "Entropy production in systems with unidirectional transitions"; Busiello, Hidalgo, and Maritan, "Entropy production for coarse-grained dynamics".

Perfect resetting "cost"[6]



$$\partial_t p(x,t) = -\partial_x j(x,t) - rp(x,t) + r\delta(x)$$

where

$$j(x,t) = -D\partial_x p(x,t) - V'(x)p(x,t)$$

In steady state

$$j_{\rm st}(x) = -D\partial_x p_{\rm st}(x) - V'(x)p_{\rm st}(x)$$

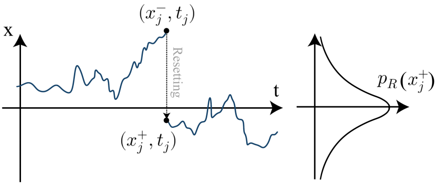
$$\langle \dot{S}_{\rm tot} \rangle \to \frac{1}{D}\int_{-\infty}^{\infty} dx\, \frac{j_{\rm st}^2(x)}{p_{\rm st}(x)}$$

$$\langle \dot{S}_{\rm rst} \rangle \to r \int p(x) \log \frac{p^s(x)}{p^s(x_0)} dx$$

[6] Fuchs, Goldt, and Seifert, "Stochastic thermodynamics of resetting".

Perfect vs. non-perfect resetting "cost"[8]

Model[7]



$$\partial_t p(x,t) = -\partial_x j(x,t) - rp(x,t) + rp_R(x)$$

$$\langle \dot{S}_{\text{local}} \rangle \to \frac{1}{D} \int_{-\infty}^{\infty} dx \, \frac{j_{\text{st}}^2(x)}{p_{\text{st}}(x)}$$

$$\langle \dot{S}_R \rangle = r \int_{-\infty}^{\infty} dx^- \int_{-\infty}^{\infty} dx^+ \, p_{\text{st}}(x^-) p_R(x^+) \log \left[\frac{p_R(x^+)}{p_R(x^-)} \right]$$

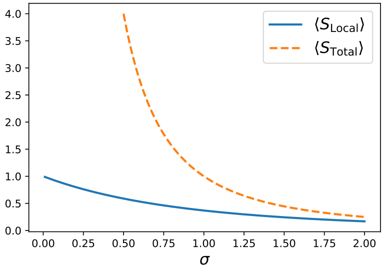
$$\langle \dot{S}_{\text{tot}} \rangle - \langle \dot{S}_{\text{Local}} \rangle = r D_{\text{KL}} \left[p_{\text{st}} || p_R \right] + r D_{\text{KL}} \left[p_R || p_{\text{st}} \right]$$

$$\Rightarrow \langle \dot{S}_{\text{tot}} \rangle \geq \langle \dot{S}_{\text{Local}} \rangle$$

[7] Evans, Majumdar, and Schehr, "Stochastic resetting and applications"; Toledo-Marin and Boyer, "First passage time and information of a one-dimensional Brownian particle with stochastic resetting to random positions"; González, Riascos, and Boyer, "Diffusive transport on networks with stochastic resetting to multiple nodes"; Dahlenburg et al., "Stochastic resetting by a random amplitude"; Besga et al., "Optimal mean first-passage time for a Brownian searcher subjected to resetting: experimental and theoretical results"; Faisant et al., "Optimal mean first-passage time of a Brownian searcher with resetting in one and two dimensions: experiments, theory and numerical tests".

[8] Mori, Olsen, and Krishnamurthy, "Entropy production of resetting processes".

Perfect vs. non-perfect resetting "cost"[9]

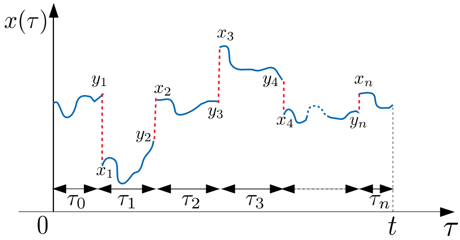


Resetting distribution $p_R(x) = e^{-x^2/(2\sigma^2)}/\sqrt{2\pi\sigma^2}$ for a system with $D = r = 1$ and $V(x) = 0$.

$$\langle \dot{S}_{\mathrm{tot}} \rangle - \langle \dot{S}_{\mathrm{Local}} \rangle = r D_{\mathrm{KL}}\left[p_{\mathrm{st}} || p_R\right] + r D_{\mathrm{KL}}\left[p_R || p_{\mathrm{st}}\right]$$

[9] Mori, Olsen, and Krishnamurthy, "Entropy production of resetting processes".

Distribution of ΔS_{tot}[10]



$$S_{\text{Total}} \approx \sum_{i=1}^{n-1} s_i$$

where

$$s_i = \log\left(\frac{p_R(x_i)}{p_R(y_{i+1})}\right) - \frac{V(y_{i+1}) - V(x_i)}{T}.$$

$$P(S_{\text{Total}}|t) \approx \frac{1}{2\pi i}\int_{\Gamma_1} dq \, \frac{1}{2\pi i}\int_{\Gamma_2} d\lambda \, e^{qS_{\text{Total}}+\lambda t} \frac{\tilde{p}(q|r+\lambda)}{1 - r\tilde{p}(q|r+\lambda)},$$

In the case of free diffusion ($V(x) = 0$):

$$\tilde{p}(q|\lambda) = \int_{-\infty}^{\infty} dx \, p_R(x) \int_{-\infty}^{\infty} dy \, \frac{1}{2\sqrt{\lambda}} e^{-\sqrt{\lambda}|x-y|} \, \exp\left[-q\log\left(\frac{p_R(x)}{p_R(y)}\right)\right].$$

[10] Mori, Olsen, and Krishnamurthy, “Entropy production of resetting processes”.

Free diffusion + p_R[11]

Exponetial resetting distribution

$$p_R(x) = \frac{1}{2a} e^{-|x|/a} .$$

$$\langle \dot{S}_{\text{total}} \rangle = \frac{D}{a(ar + \sqrt{Dr})} r .$$

$$p(s) = \begin{cases} \frac{2}{3} e^{-s} & \text{for } s > 0 , \\ \frac{2}{3} e^{2s} & \text{for } s < 0 . \end{cases}$$

Gaussian resetting distribution

$$p_R(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/(2\sigma^2)} .$$

$$\langle S_{\text{total}} \rangle = \frac{D}{\sigma^2} t$$

$$p(s) \approx \sqrt{\frac{e}{2s}} e^{-\sqrt{2s}} .$$

[11] Mori, Olsen, and Krishnamurthy, "Entropy production of resetting processes".

Intermittent potentials

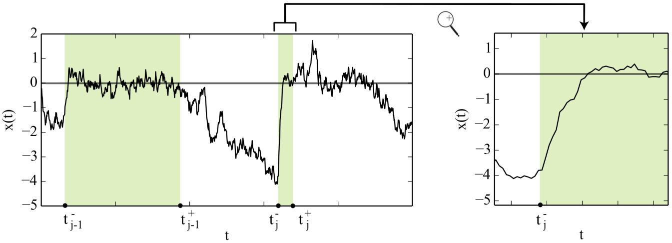
Scheme that mimicks realistic resetting:

i) Turn on a confining potential $V(x)$ for a duration $\tau \in \psi_{\text{on}}(\tau)$.

ii) Let the potential be turned off a duration $\tau \in \psi_{\text{off}}(\tau)$.

iii) Return to step i).

Several past studies exist[12][13][14][15][16][17].



[12] Mercado-Vásquez et al., “Intermittent resetting potentials”.

[13] Santra, Das, and Nath, “Brownian motion under intermittent harmonic potentials”.

[14] Gupta et al., “Stochastic resetting with stochastic returns using external trap”.

[15] Gupta, Pal, and Kundu, “Resetting with stochastic return through linear confining potential”.

[16] Alston, Cocconi, and Bertrand, “Non-equilibrium thermodynamics of diffusion in fluctuating potentials”.

[17] Gupta and Plata, “Work fluctuations for diffusion dynamics submitted to stochastic return”.

How long should the confining potential be turned on?

- Finite-time resetting: enough time for equilibriation[a]
- Finite-time resetting: enough time for First passage to trap minimum[b]
- Finite-time resetting: switch-off time of intermittent potential[c]
- Finite-time resetting: Deterministic returns[d] or refractory periods[e].

[a] Besga et al., "Optimal mean first-passage time for a Brownian searcher subjected to resetting: experimental and theoretical results".

[b] Gupta et al., "Stochastic resetting with stochastic returns using external trap".

[c] Mercado-Vásquez et al., "Intermittent resetting potentials".

[d] Tal-Friedman et al., "Experimental realization of diffusion with stochastic resetting".

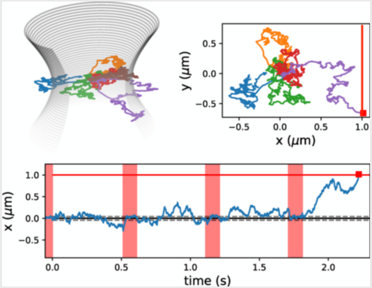
[e] Rotbart, Reuveni, and Urbakh, "Michaelis-Menten reaction scheme as a unified approach towards the optimal restart problem".

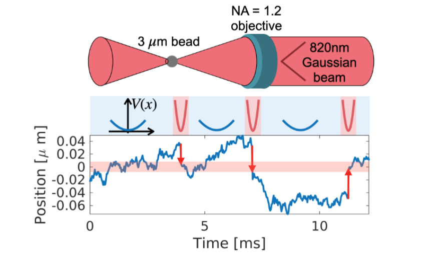
Connection with $p_R(x)$

$$p_R(x) = \frac{1}{2a} e^{-|x|/a} \rightarrow \text{V-shaped confining potential}$$
$$p_R(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/(2\sigma^2)} \rightarrow \text{Harmonic confining potential}$$

If resetting is implemented by allowing equilibriation in the resetting potential.

Recent Experimental setups: Resetting via equilibriation





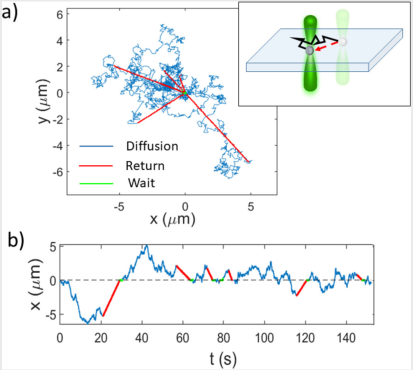
Optimal mean first-passage time for a Brownian searcher subjected to resetting: Experimental and theoretical results

Benjamin Besga, Alfred Bovon, Artyom Petrosyan, Satya N. Majumdar, and Sergio Ciliberto
Phys. Rev. Research **2**, 032029(R) – Published 30 July 2020

Experimental test of Landauer's principle for stochastic resetting

Rémi Goerlich,[1,2,*] Minghao Li,[3,4] Luis B. Pires,[2] Paul-Antoine Hervieux,[1] Giovanni Manfredi,[1] and Cyriaque Genet[2]
[1]*Université de Strasbourg, CNRS, Institut de Physique et Chimie des Matériaux de Strasbourg, UMR 7504, F-67000 Strasbourg, France*
[2]*Université de Strasbourg, CNRS, Centre Européen de Sciences Quantiques & Institut de Science et d'Ingénierie Supramoléculaires, UMR 7006, F-67000 Strasbourg, France*
[3]*Department of Physics, University of Basel, Klingelbergstrasse 82, 4056 Basel, Switzerland*
[4]*Swiss Nanoscience Institute, Klingelbergstrasse 82, 4056 Basel, Switzerland*
(Dated: June 19, 2023)

Other Experimental setups: resetting via deterministic returns



Experimental Realization of Diffusion with Stochastic Resetting

Ofir Tal-Friedman, Arnab Pal, Amandeep Sekhon, Shlomi Reuveni, and Yael Roichman
The Journal of Physical Chemistry Letters **2020** *11* (17), 7350-7355
DOI: 10.1021/acs.jpclett.0c02122

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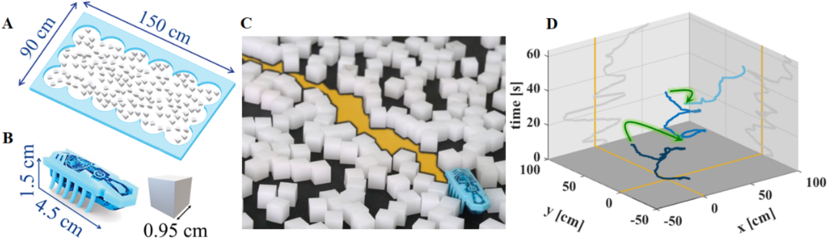


Environmental memory facilitates search with home return

Amy Altshuler,[1,2] Ofek Lauber Bonomo,[1,2] Nicole Gorohovsky,[3] Shany Marchini,[3]
Eran Rosen,[1] Ofir Tal-Friedman,[4] Shlomi Reuveni,[1,2,*] and Yael Roichman[1,2,4,†]

[1] *The Raymond and Beverley School of Chemistry, Tel Aviv University, Tel Aviv 6997801, Israel.*
[2] *Center for Physics and Chemistry of Living Systems, Tel Aviv University, Tel Aviv, 6997801, Israel.*
[3] *Department of Materials Science and Engineering, Tel Aviv University, Tel Aviv, 6997801, Israel.*
[4] *The Raymond and Beverley School of Physics & Astronomy, Tel Aviv University, Tel Aviv 6997801, Israel.*

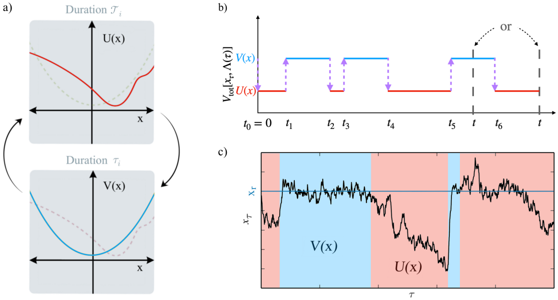
Work done for implementing resetting

Resetting schemes[a]

$$\frac{dx_\tau}{d\tau} = -\gamma^{-1}\nabla V_{\text{tot}}[x_\tau, \Lambda(\tau)] + \eta(\tau)\,,$$

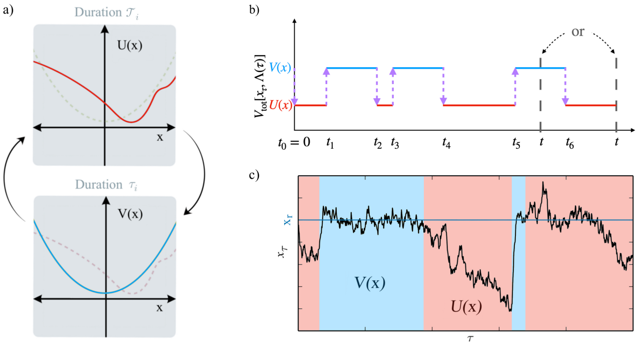
$$W = \int_0^t \frac{\partial V_{\text{tot}}}{\partial \Lambda}\frac{d\Lambda}{dt'}dt'$$

[a] Gupta, Plata, and Pal, "Work fluctuations and Jarzynski equality in stochastic resetting"; Gupta and Plata, "Work fluctuations for diffusion dynamics submitted to stochastic return"; Goerlich et al., *Experimental test of Landauer's principle for stochastic resetting*; Pal et al., "Thermodynamic trade-off relation for first passage time in resetting processes".



The work done to reset the system is the net energy change at the instances $U(x) \leftrightarrow V(x)$

$$w[\{x_\tau\}_0^t] \equiv \sum_{j=1} [U(x^+_{2j-2}) - V(x^-_{2j-2})] + \sum_{j=1} [V(x^+_{2j-1}) - U(x^-_{2j-1})]$$



[18] Olsen et al., *Thermodynamic cost of finite-time stochastic resetting*.

Heat dissipated while resetting[19]

$$q[\{x_\tau\}_0^t] \equiv \sum_{j=1}[U(x_{2j-2}^+) - U(x_{2j-1}^-)] + \sum_{j=1}[V(x_{2j-1}^+) - V(x_{2j}^-)]$$
$$+ \begin{cases} V(x_N^+) - V(x(t)) & \text{for } x(t) \in \text{return phase}\ , \\ U(x_N^+) - U(x(t)) & \text{for } x(t) \in \text{exploration phase}\ , \end{cases}$$

[19] Olsen et al., *Thermodynamic cost of finite-time stochastic resetting*.

Heat dissipated while resetting[19]

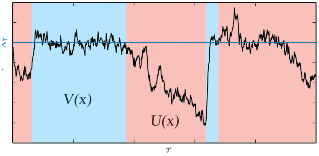
$$q[\{x_\tau\}_0^t] \equiv \sum_{j=1}[U(x_{2j-2}^+) - U(x_{2j-1}^-)] + \sum_{j=1}[V(x_{2j-1}^+) - V(x_{2j}^-)]$$

$$+ \begin{cases} V(x_N^+) - V(x(t)) & \text{for } x(t) \in \text{return phase}\ , \\ U(x_N^+) - U(x(t)) & \text{for } x(t) \in \text{exploration phase}\ , \end{cases}$$

$$\Delta E[\{x_\tau\}_0^t] = w[\{x_\tau\}_0^t] - q[\{x_\tau\}_0^t]$$

where

$$\Delta E[\{x_\tau\}_0^t] = -V(x_0^-) + \begin{cases} V(x(t)) & \text{for } x(t) \in \text{return phase}\ , \\ U(x(t)) & \text{for } x(t) \in \text{exploration phase}\ , \end{cases}$$

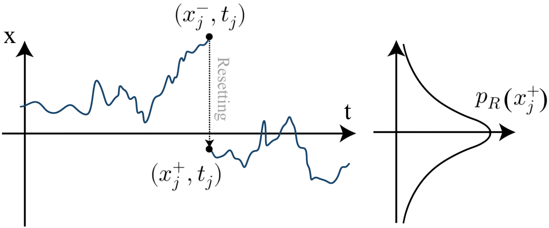


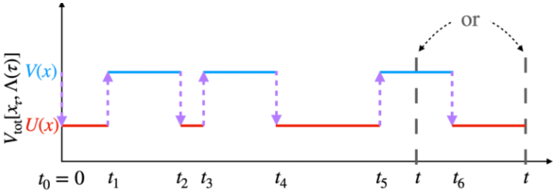
[19] Olsen et al., *Thermodynamic cost of finite-time stochastic resetting*.

Heat dissipated while resetting

$$q[\{x_\tau\}_0^t] \equiv \sum_{j=1} [U(x^+_{2j-2}) - U(x^-_{2j-1})] + \sum_{j=1} [V(x^+_{2j-1}) - V(x^-_{2j})]$$

$$q[\{x_\tau\}_0^t] \equiv \sum_{j=1} [U(x^+_{2j-2}) - U(x^-_{2j-1})] + \sum_{j=1} \log \frac{p_R(x^+_{2j})}{p_R(x^+_{2j-1})}$$

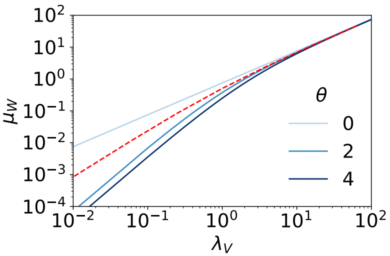




$$C(k,t) \equiv \left\langle e^{kW} \right\rangle_t = \sum_{n=0}^{\infty} \int_{-\infty}^{+\infty} \mathrm{dW}\, \mathcal{P}_\mathrm{t}(\mathrm{W},\mathrm{n})\, \mathrm{e}^{\mathrm{kW}} = \sum_{\mathrm{n}=0}^{\infty} \psi_\mathrm{n}(\mathrm{k},\mathrm{t})$$

$$\begin{aligned}\tilde{C}(k,s) &= \tilde{\psi}_0(k,s) + \sum_{n=1}^{\infty} \tilde{\psi}_n(k,s) \\ &= \frac{\tilde{\psi}_0(k,s) + \tilde{H}(s)\, \tilde{I}(k,s)}{1 - \tilde{h}(s)\, \tilde{I}(k,s)}\,.\end{aligned}$$

[20] Olsen et al., *Thermodynamic cost of finite-time stochastic resetting*.



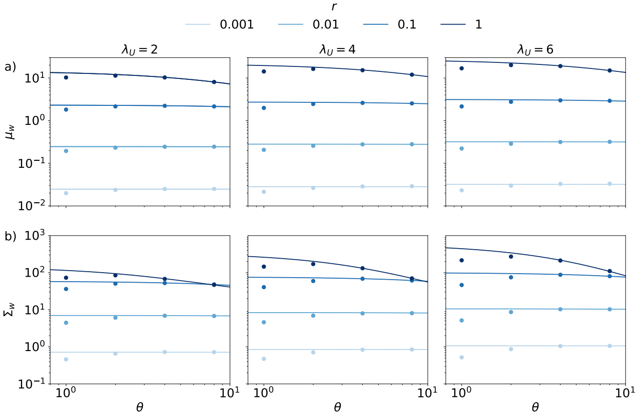
	Harmonic potential ($V = \frac{1}{2}\lambda_V x^2$)	V-shaped potential ($V = \gamma_V \lvert x \rvert$)
Finite-time	$\frac{D\lambda_V}{1 + r\theta\tau_{\text{rel}}}$	$\frac{1}{1 + r\theta\tau_{\text{rel}}} \frac{4D\sqrt{r}\alpha_V}{\sqrt{r} + 2\sqrt{\alpha_V}}$
IDR[21]	$D\lambda_V$	$\frac{4D\sqrt{r}\alpha_V}{\sqrt{r} + 2\sqrt{\alpha_V}}$
FPR[22]	See Fig.	$\frac{4D\sqrt{r}\alpha_V}{\sqrt{r} + 2\sqrt{\alpha_V}}$

[21] Mori, Olsen, and Krishnamurthy, "Entropy production of resetting processes".

[22] Gupta and Plata, "Work fluctuations for diffusion dynamics submitted to stochastic return".

[23] Olsen et al., *Thermodynamic cost of finite-time stochastic resetting*.

Equilibriation condition[24]



- Mean work can also be computed for arbitrary durations of either phase.

[24] Olsen et al., *Thermodynamic cost of finite-time stochastic resetting*.

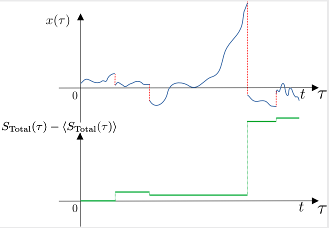
Condensation transition[25]

Gaussian resetting distribution

$$p_R(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/(2\sigma^2)}$$

$$p(s) \approx \sqrt{\frac{e}{2s}} e^{-\sqrt{2s}}$$

$\Rightarrow$ S_{tot} rate function shows a condensation transition



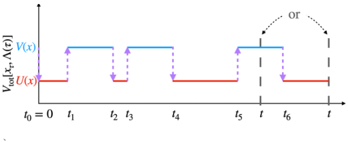
[25] Mori, Olsen, and Krishnamurthy, "Entropy production of resetting processes".

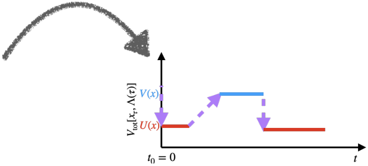
Open Questions

- Does Condensation transition take place for exploration phase in a potential as well?
- Does it happen for the work distributions in these cases ?
- Condensation transition with absorbing boundary?
- Cost trade-off analysis[26]

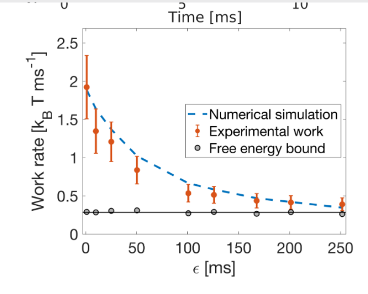
[26] Pal et al., "Thermodynamic trade-off relation for first passage time in resetting processes".

Open Questions





Lower bound on the cost of resetting like the Landauer bound for information erasure[a]



[a] Goerlich et al., *Experimental test of Landauer's principle for stochastic resetting*; Fuchs, Goldt, and Seifert, "Stochastic thermodynamics of resetting".

Thank you!

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